Carbon dating reveals Bakhshali manuscript is centuries older than scholars believed and is formed of multiple leaves nearly 500 years different in age

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Introduction:

The Bakhshali manuscript, an early Indian mathematical treatise housed at the University of Oxford's Bodleian Libraries, has for the first time been radiocarbon dated. The results reveal that this very rare birch bark manuscript dates from more than four centuries earlier than scholars previously believed. Previous dating of this ancient mathematical record had been estimated based on the style of writing and the literary and mathematical content.

The dating of the manuscript has long been the subject of academic debate but there has never before been any consensus on its exact age. Scholarly estimates ranged dramatically from 200 CE to 1100 CE Now, new carbon dating results produced by a team of researchers at the University of Oxford[1] are able to shed light on why it has previously been so difficult for scholars to pinpoint the manuscript's date. It is actually composed of material from at least three dates, with some pages dating from as early as the 3rd to 4th century and others dating from the 8th and 10th centuries.

The date of the manuscript was particularly important to determine because of the Bakhshali manuscript's significance in the early history of mathematics. The manuscript contains one of the earliest uses of zero (written as a dot in this manuscript) as a placeholder, i.e. the use of zero to indicate orders of magnitude in a number system.

Background on the manuscript:

The Bakhshali manuscript, which consists of 70 leaves of birch bark, is a manuscript of Indian mathematics that was discovered in 1881 in a region of what is now Pakistan, not far from the site of the ancient trading centre of Taxila.

The most authoritative treatise on it is the Brown University Ph.D. thesis of Takao Hayashi, now on the faculty of Doshisha University in Japan. The text's structure consists of rules and sample problems in verse, with a commentary explaining them. It is not clear whether the rules and sample problems and the commentary are the work of a single author or not. Hayashi dated the content of the manuscript to the 7th century due to the similarities of its style and vocabulary with that of other medieval mathematical works (Hayashi 1995, 148–9). It is likely that the text was then copied in manuscripts and Hayashi asserted that the oldest extant copy of the text, housed at the Bodleian Libraries, dated from a period ranging from the 8th to the 12th century (Hayashi 1995, 23–25).

In early 2017, a team of researchers from the University of Oxford's Bodleian Libraries and the Oxford Radiocarbon Accelerator Unit collaborated on a project to carbon date this remarkable manuscript. After lengthy discussions, and based on stylistic grounds, the team decided that it was not possible to take just one representative sample. Five samples were extracted from the manuscript, three of which were used for the C-14 analysis, with the remaining two samples being kept intact. This process, including the location where the samples were extracted from, is fully documented for future reference. The results reveal that the three samples tested date from three different centuries, one (Folio 16) from 224-383 AD, another (Folio 17) from 680-779 AD and another (Folio 33) from 885-993 AD.

[1] The research team included: David Howell, Head of Heritage Science at the Bodleian Libraries, Gillian Evison, Head of the Bodleian Libraries Oriental Section, Camillo Formigatto, the John Clay Sanskrit Librarian at the Bodleian Libraries, Virginia Llado-Buisan, Head of Conservation at the Bodleian Libraries and David Chivall, Chemistry Laboratory Manager at the Oxford Radiocarbon Accelerator Unit.

Background on radiocarbon dating:

Radiocarbon dating is one of the most widely used scientific dating methods in archeology and environmental science and can be applied using most organic material formed during the last 50,000 years. Radiocarbon is a naturally occurring radioactive isotope of the element carbon and is continually produced in the upper atmosphere due to cosmic rays striking the earth. It is quickly mixed into carbon dioxide and distributed throughout the atmosphere and oceans. Plants and trees then incorporate the carbon dioxide via photosynthesis and use the carbon to build structures, such as those that form the birch bark on which the Bakhshali manuscript was written. As soon as this carbon is locked into the structure of the bark, it is isolated from the atmosphere and no new radiocarbon can be added. Radiocarbon decays with a half-life of 5,730 years, so once the bark is formed the amount of radiocarbon within it decreases but the amount of stable carbon in the bark remains constant. By measuring the ratio of radiocarbon to stable carbon in the bark it is possible to calculate how long ago the bark was formed.

Of course, there are some complications involved in radiocarbon dating, two of which are variations in the atmospheric concentration of radiocarbon and the possible contamination of the material to be dated.

Variations in the atmospheric concentration of radiocarbon can be caused, among other sources, by a change in the production rate of radiocarbon in the upper atmosphere or changes in ocean circulation. These atmospheric variations can affect the radiocarbon age of a material, for example if organic material was formed during a period of low atmospheric radiocarbon concentration it may appear no younger than older material formed during a period of high atmospheric concentration. To deal with this complication, radiocarbon measurements are calibrated against material of known age (such as tree rings dated by dendrochronology) to give calibrated age ranges, which are usually presented as 95% confidence intervals.

The easiest way to deal with potential contamination is through careful sampling. For this reason areas of ink in the Bakhshali manuscript were avoided when taking samples, because the ink will likely contain carbon of a different age to the bark (and the age of the carbon in the ink would very likely not correspond with the date that the manuscript was written). Beyond this, material to be radiocarbon dated routinely undergoes chemical pretreatment to remove as much potential contamination as possible without destroying the sample. The Bakhshali samples were very small (1.4 to 1.8 mg of bark) so were treated with a mild chemical pretreatment, which included washes using organic solvents to remove any oils or grease on the manuscript and a sequential acid-base-acid wash to remove any potential carbonates or humics.

Methodology:

Three samples of bark (*Betula sp.*) from three different folios of the Bakhshali manuscript were used (Folio 16, 1.83 mg; Folio 17, 1.51 mg; Folio 33, 1.39 mg). The bark was pretreated with sequential washes of organic solvents (acetone, 50°C, 1 hour; methanol, 50°C, 1 hour; chloroform, room temperature, 1 hour) and three rinses of ethanol and four of ultrapure water, before sequential washes of 1 M hydrochloric acid, 0.2 M sodium hydroxide and 1 M hydrochloric acid. After each acid and base wash the samples were rinsed three times with ultrapure water. (ORAU pretreatment code VV*; Brock et al., 2010).

After chemical pretreatment the bark samples were converted to carbon dioxide using an elemental analyzer, the carbon dioxide was graphitized at 560°C in the presence of hydrogen gas and an iron catalyst (Dee and Bronk Ramsey, 2000) and the radiocarbon content of the graphite determined using the Oxford accelerator mass spectrometer (Bronk Ramsey et al., 2004). Radiocarbon measurements were calibrated against the IntCal 13 curve (Reimer, 2013) using OxCal v4.3.2 (Bronk Ramsey, 2009).

Results:

Radiocarbon determinations and calibrated age ranges are shown in Table 1 and Figures 1 to 3.

Table 1: Laboratory codes, radiocarbon determinations, stable carbon isotope compositioin and calibrated ages for the birch bark from the Bakhshali manuscript. Presented uncertainties in the radiocarbon determination are one standard deviation.

Sample	ORAU laboratory code	Radiocarbon determination / BP	$\delta^{13}C$ / ‰	Calibrated age, 95.4% confidence interval / cal AD
Bakhshali folio 16	OxA-35,405	1751 ± 29	-27.3	224 – 383
Bakhshali folio 17	OxA-35,406	1247 ± 27	-27.0	680 – 868
Bakhshali folio 33	OxA-35,407	1108 ± 26	-24.6	885 – 993

Figure 1. Radiocarbon determination and 95.4% calibrated age range for folio 16.

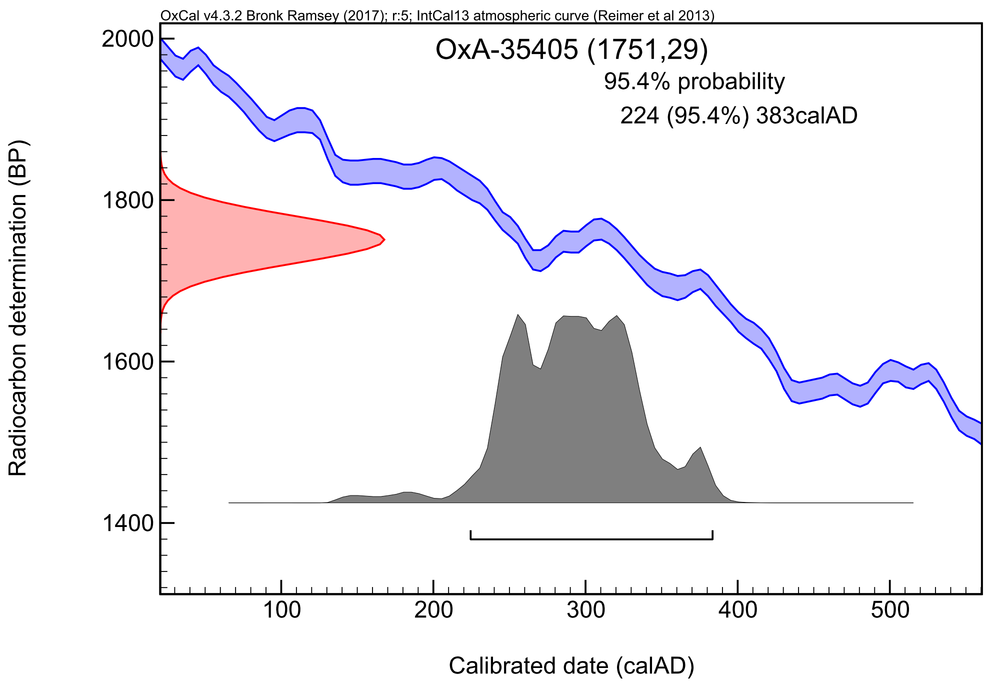


Figure 2. Radiocarbon determination and 95.4% calibrated age range for folio 17.

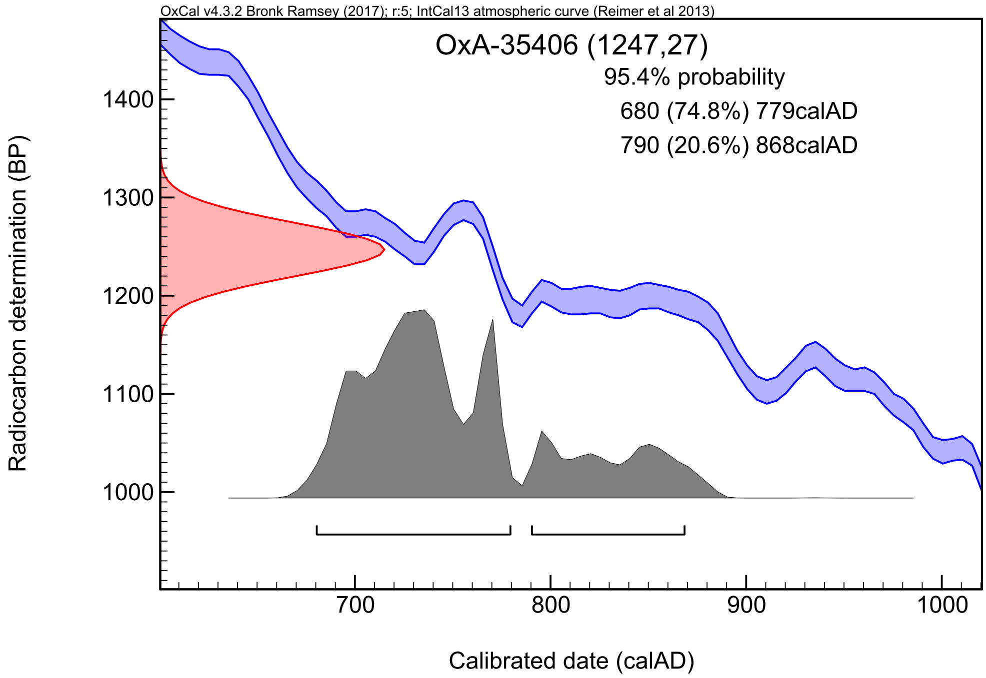
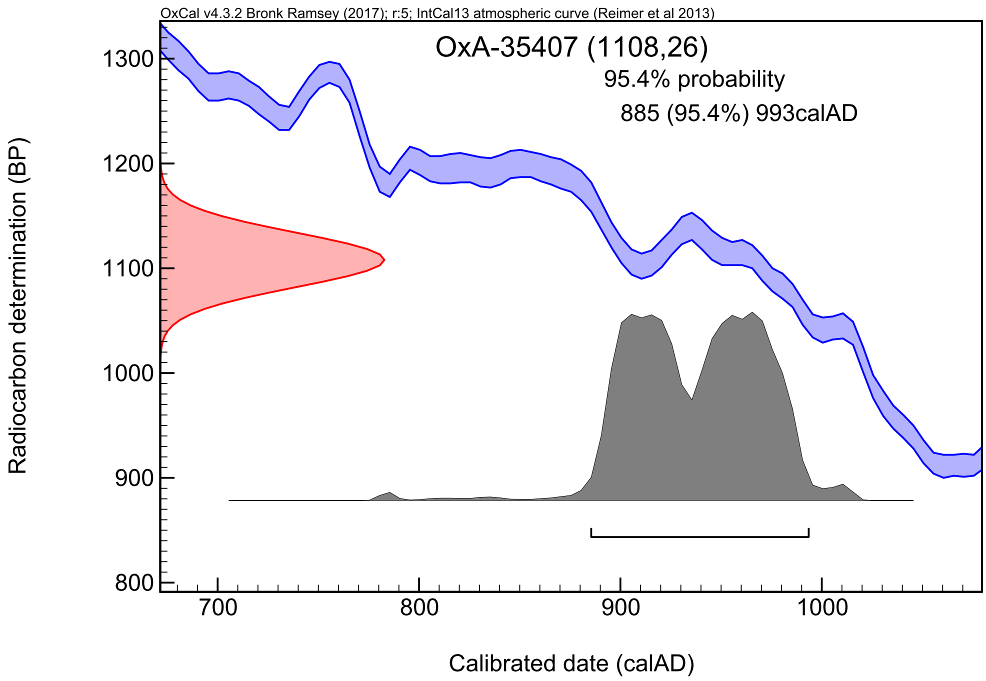


Figure 3. Radiocarbon determination and 95.4% calibrated age range for folio 33.



Discussion:

Significance of the results for the history of mathematics and the birth of zero – by Oxford University Professor of Mathematics Marcus du Sautoy:

The Bakhshali manuscript is significant for the history of mathematics because it contains numerous dots representing nothing or zero. Until now, it was never known how old this zero was.

The zero used in the Bakhshali manuscript is not yet a number in its own right. It is a place holder being used as part of another number written in the place number system. We write 101 to indicate 1 hundred, no tens and 1 unit. The zero is indicating the absence of 10s. This idea of needing a symbol to represent nothing as part of writing numbers in the place number system has a very ancient heritage. 5000 years ago, the Babylonians used a double wedge for nothing as part of the cuneiform symbols to write numbers on stone tablets. The Mayans were using a symbol of a shell to denote absence in their number system 2000 years ago.

So the dot used in the Bakhshali manuscript cannot claim to be the first use of zero as part of building larger numbers (as a placeholder). But why it is so exciting is that this zero used in India, represented by a dot, is the seed from which the concept of zero as a number in its own right emerged some centuries later, something many regard as one of the of the great moments in the history of mathematics. In other words, it was the dot that we see in the Bakhshali manuscript that went onto become the symbol that was first used for zero as a number in its own right.

The idea of creating a symbol to represent nothing is one of the great mathematical breakthroughs in the history of mathematics. The creation of zero as a number in its own right, not as a mere placeholder, represented by a circle or a dot happened in India where contemplating the void was part of the philosophy of that region. The idea of zero as a number first appeared in the astronomer Brahmagupta's text, Brahmasphutasiddhanta, which was written in 628 CE and is the first text to talk about zero as a number in its own right and to include a discussion of the arithmetic of zero.

The Bakhshali manuscript helps to illustrate how vibrant mathematics has been in India and the east for centuries. It is also testament to the way mathematics crosses cultural, historical and political boundaries. The concept of zero is used across the globe. It is a key ingredient of the digital world. Yet we now know that it was mathematicians in India in 200-400 CE who planted the seed of the idea that would become so fundamental to the modern world.

Significance of the results for South Asian cultural history – by Camillo Formigatti, John Clay Sanskrit Librarian at the Bodleian Libraries:

The carbon dating has revealed that the Bakhshali manuscript is a composite document and consists of at least three different parts with different ages. Due to the fragmentary nature of the manuscript, it is difficult to reconstruct the original order of the folios and consequently of the extant texts. Until now, the manuscript has been studied as if it were one single item. The first editor of the manuscript, G. R. Kaye, employed the following criteria to establish the order of the folios:

(1) Logical sequence of contents.
(2) The 'find order.'
(3) Physical appearance such as the size, shape, degree of damage, and knots in the birch bark.
(4) The script and language.
(5) Numbered sūtras.

In the most recent complete study of the manuscript (1995), Takao Hayashi does not deem the fourth criterion reliable enough for the reconstruction of the order of the folios. This approach is again based on the assumption that all extant parts of the manuscript were written at the same time – an assertion now overturned by the new carbon dating results.

The early date of folio 16 fits well into the cultural milieu in which this part of the manuscript was probably produced and circulated. The content of the Bakhshali manuscript is similar to the type of texts that Buddhist merchants would have needed to study (and possibly use as reference) for their daily trading activities. It includes very practical mathematical examples and equations, such as how to compute the loss in weight of a quantity of impure metal in the process of refining it, etc.

The manuscript was recovered in the village of Bakhshali in Pakistan, in an area that belongs to the historical Gandhāra region. It is a region in which major cities such as Peshawar (Skt. Puruṣapura) and Taxila (Skt. Takṣaśilā) were important commercial and cultural hubs. This area belonged to the Indian, Persian, and Greek cultural spheres of influence and had contacts with Chinese culture through the Silk Road. The oldest Buddhist and South Asian manuscripts extant were also found in the Greater Gandhāra region and their dates range from the first century BCE to the fourth century CE – so a similar time period to that of folio 16 of the Bakhshali manuscript. Although written in a different script and a different language than the Bakhshali manuscript, these Buddhist manuscripts are also written on birch-bark.

Moreover, the language used in the manuscript is not standard Sanskrit, and according to Hayashi it shows features of what is called Buddhist Hybrid Sanskrit, as well as of other Middle-Indo Aryan languages (Prakrit and Apabhraṃśa), and also of Old Kashmiri. This is yet another feature that can be explained by the fact that the manuscript in its present state is composed of at least three different manuscripts with different dates. The three dates would then correspond to different stages of linguistic development. This assumption however has to be corroborated by a closer linguistic study of the manuscript. The carbon dating results are likely to help scholars to better reconstruct the original structure of the different manuscripts and the order of the folios, and hopefully to be able to make a more informed assessment of the different textual layers.

Additional note:

The Bakhshali manuscript is the sole surviving copy of a mathematical text or texts. Carbon dating has revealed it to be at least three manuscripts gathered together into one manuscript. It is possible that the Bakhshali manuscript is made up of more than one text. More research is needed to better understand what the manuscript consists of.

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