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Comment on "Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture"

Emil Makovicky

Lu and Steinhardt (Reports, 23 February 2007, p. 1106) claimed the discovery of a large, potentially quasi-crystalline Islamic tiling in the Darb-i Imam shrine but regard the earlier Maragha tiling, previously described as quasiperiodic, as a small isolated motif. We demonstrate that the Darb-i Imam pattern is periodic and that the quasi-crystalline discs superimposed on its lattice are derivatives of the Maragha pattern.

When did quasi-crystalline patterns appear in Islamic architecture? Lu and Steinhardt (*1*, *2*) reported the discovery of quasi-crystalline tiling patterns on the Darb-i Imam shrine, Isfahan, Iran (1453 C.E.), yet these patterns are not profoundly different from the previously described patterns of the Gunbad-i Kabud tomb tower in Maragha, Iran (1197 C.E.) (*3*). At both localities, Islamic artists created decagonal/pentagonal quasiperiodic patterns starting from a central point and developed them as a cartwheel pattern with tenfold point group symmetry. In the patterns of the Darb-i Imam shrine (*1*, *4*), such quasiperiodic discs were placed in the (½,0) and (0,½) nodes of a periodic, centered net (continuous lines in Fig. 1) with plane group symmetry cmm [perpendicular reflections and half-turns; see also figures 3C and S7A in (*2*)]. The Maragha pattern is a quasiperiodic cartwheel pattern (*3*) spread over two adjacent panels of the building, with the origin at an upper corner of the building (Fig. 2A). A full 180-degree segment of a quasi-crystalline cartwheel pattern is always present (*1*). Its repetition around the eight-sided building is an architectural rather than a geometric problem. The half-cartwheel present at Maragha consists of over 400 tiles, whereas the Darb-i Imam half-cartwheel contains about 6% more tiles at its most optimistic delineation. Therefore, these cartwheel discs are large enough to demonstrate their quasi-crystalline character. Pattern adjustments along the contact of two adjacent cartwheel patterns are the only departures from quasi-periodicity in the Maragha patterns (Fig. 2A).

In the Darb-i Imam pattern, the space between four adjacent cartwheel discs with radial symmetry (each of them situated in the interior of a large-scale 10-fold star in Fig. 1) is occupied by 3 × 2 periods of a small-scale cmm pattern. Variability of infill of the small 10-fold stars present in these portions is discussed further below. A lozenge-like configuration of the large-scale net is placed over these portions. Fusing of these periodic portions with the cartwheel portions proceeds by overlapping pattern portions, which are common to both patterns. Details of this perfect fusion were lost in the transcription to dart-and-kite tiles performed by Lu and Steinhardt (*1*), leading them to believe that they do not deal with an overall periodic pattern (2). However, the large-scale line framework of the Darb-i Imam pattern (Fig. 1), in the polygons of which the tiles are embedded (*1*, *4*), is the most common periodic cmm pattern of Seljuk art (*5*). The extent of the area of the cmm pattern worked out by the artisans (Fig. 1) precludes any other interpretation and therefore the pattern, as constructed, cannot be extended as a quasi-periodic pattern. Placing of cartwheel discs with 10-fold symmetry in the positions that display only twofold symmetry in the plane group does not change this situation. Twinning of the pattern was not observed. The large-scale cmm pattern is periodic and thus it does not possess the pattern inflation/deflation property. As a consequence, the Darb-i Imam pattern is not self-similar in a strict sense. Further examples of cmm mosaics nearly identical to the one discussed occur both at the Darb-i Imam and Friday Mosque in Esfahan.

All of these tilings use the same basic "Maragha-type" tiles: pentagons, butterflies, and rhombuses with marked acute vertices. Interconversion (i.e., mapping) between Maragha-type tiles and Penrose tiles was described in (*3*). When we redraw the Maragha pattern using the elongated hexagon and hourglass tiles of a so-called M2 tiling of (*3*) [see figures 9 and 10 in (*3*)], which are analogous to the two principal tiles of the "girih" set described in (*1*), we obtain a system of overlapping decagons. Each of these encloses a 10-fold star filled by Maragha tiles (Fig. 2A). In selected positions, these filled stars can be rotated by $n \times 72°$ without interrupting the line linkages of the pattern, as well as flipped or even emptied of the contents. Together with similar, linkage-preserving rotations of selected pentagons filled by basic tiles, this leads to pattern variations (high-entropy pattern variants with all linkages between clusters preserved) and to the composite elements which possess 5- or 10-fold rotational symmetry, such as the ornamental stars and pentagons observed at Maragha (Fig. 2A) and the 'blank stars' of the Darb-i Imam pattern (Fig. 2B) (these were used also in the periodic portions of the latter pattern).

Detailed tile tracing (Fig. 2) reveals that the quasi-crystalline cartwheel discs at the positions (½,0) and (0,½) of the Darb-i Imam pattern, and the cartwheel that forms the Maragha pattern, are

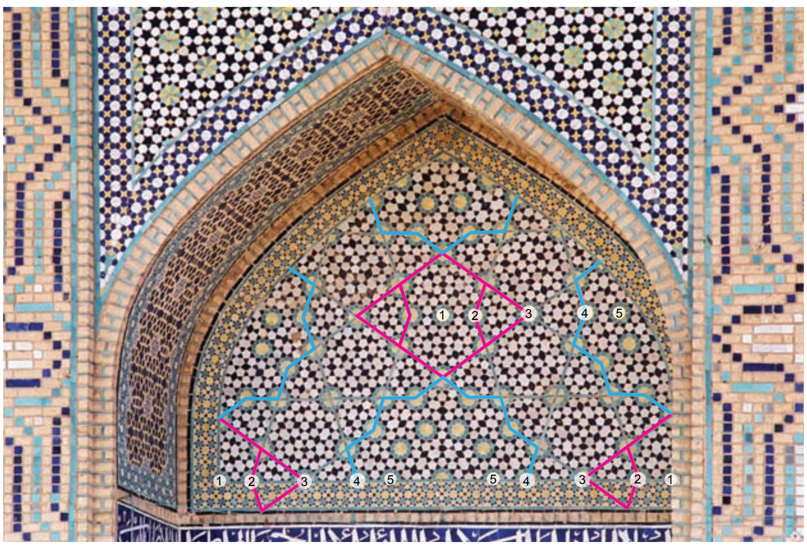

Fig. 1. Portal of the Darb-i Imam Mosque (1453) in Esfahan. Translation identity of selected elements of the large-scale cmm net is indicated by coloring [large 10-fold stars in Wyckoff positions (½,0) are green, lozenges in (0,0) are red], whereas that of selected small 10-fold stars is shown by correspondence of inscribed numbers. The tympanum contains one unit mesh of the cmm pattern, with the origin in the lower left-hand corner and two upper nodes truncated.

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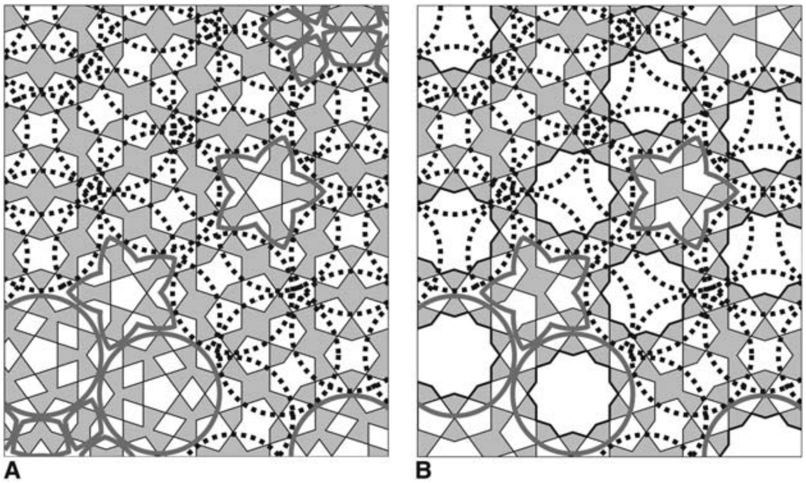

Fig. 2. (**A**) A panel of Maragha tiling colored in Darb-i Imam style. (**B**) A corresponding portion of the Darb-i Imam cartwheel pattern. A quarter-cartwheel of each tiling is shown, with the origin always in the upper left corner of the respective panel. A vertical contact with the adjacent cartwheel runs just inside of the right-hand edge of (A). Eye-leading circles based upon M2 tiling of (*1*) indicate some of the ten-fold star outlines of Maragha tiling in (A); a portion of them were replaced by blank stars in (B). Individual elements of, and relations between, (A) and (B) tilings are discussed in the text.

nearly identical both in tile arrangement and in size (the largest radius of these disc patterns reaches a prominent local 10-fold configuration). Both of them allow inflation/deflation inside a cartwheel and are representative fragments of quasi-crystalline decagonal cartwheel patterns, either adjusted to the architecture of the building at their periphery or embedded in a periodic framework. The Darb-i Imam cartwheel discs appear to be later variations of the tiling scheme from Maragha. This corroborates the suggestion by Makovicky (*3*), who dated the discovery of quasi-crystalline patterns to the years 1196 and 1197—much earlier than Lu and Steinhardt (*1*)—and confirms the exceptional cultural value of the Gunbad-i Kabud tower in Maragha.

In broad analogy to the present-day situation in Morocco, we believe that the artisans were satisfied by creating a large fundamental domain without being concerned with a mathematical notion of infinitely expandable quasiperiodic patterns. However, they understood and used to their advantage some of the local geometric properties of the quasi-crystalline patterns they constructed.

References and Notes

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