



Supporting Online Material for

The Advantage of Abstract Examples in Learning Math

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This PDF file includes

Materials and Methods
SOM Text
Tables S1 and S2
References
Appendix

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Mathematical Concepts and their Instantiations

Unlike everyday abstract concepts such as *love*, mathematical concepts have precise definitions based on their relational structure. For example, *derivative* of a function $f(x)$ has the following definition:

$$f'(x) \equiv \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

This definition, like all mathematical definitions, communicates the essential structural properties of the concept. To add additional information creates a specific instantiation. For example, suppose f is a distance function of time with distance units of miles and time units of hours, $f(x) = 3x^2 + 5x$, perhaps $x = 2$ hours. When more extraneous information is added (e.g. f is a function describing the distance of a particular red car), a more concrete instantiation is created.

Therefore, a mathematical concept is an abstract entity and any type of expression or embodiment of the concept is an instantiation. Instantiations that communicate minimal extraneous details, beyond the defining structural information, are generic instantiations, while those that communicate more extraneous information are concrete. For example, concrete instantiations of mathematics include physical manipulatives that might be used by elementary students as well as contextualized, real-world examples that might be encountered by more advanced learners. Generic instantiations are any that minimize extraneous detail, such as traditional symbolic notation (e.g. the statement of the definition of derivative above). Furthermore, because mathematical concepts are well-defined and because generic instantiations minimize irrelevant detail they represent mathematical concepts in a manner similar to statements of abstract rules.

In the present research, study participants learned the concept of a commutative mathematical group of order three. This concept has a set of abstract properties described below. Some participants learned a generic instantiation, while others learned one or more concrete instantiations.

METHODS









Experiment 1

Participants Eighty undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course. Students were randomly assigned to one of four conditions that specified the type and number of instantiations they learned.







Materials and Design The experiment consisted of two phases. In phase 1, participants learned one, two, or three instantiations of a mathematical concept. The instantiations were either generic or concrete. There were four between subject conditions, which specified the number and type of







instantiations learned in phase 1: Generic 1, Concrete 1, Concrete 2, or Concrete 3. Total training was equated across condition. In phase 2, participants were tested on an isomorphic transfer domain. The to-be learned concept was a commutative mathematical group of order three. This concept is defined by a set of principles. Specifically, a *Commutative Mathematical Group of Order Three* is a set of three elements, or equivalence classes, and an associated binary operation over which the following properties hold: associativity, commutativity, existence of identity, and existence of inverses. If the operation is denoted by +, then the following is true. The *Associative Property* states that for any elements, x, y, z , of the set, $(x + y) + z = x + (y + z)$. The *Commutative Property* states that for any elements x, y of the set, $x + y = y + x$. In addition, there is an element, I , in the set called the *Identity Element*, such that for any element, x , $x + I = x$. Finally, for any element, x , there exists an *Inverse Element*, y , such that $x + y = I$.

To understand such a system with arbitrary symbols would involve learning the rules presented in Table 1 (Generic instantiation). However, concrete contexts can be created in which prior knowledge and familiarity may assist learning. Four instantiations of a mathematical group were constructed: three concrete (Concrete A, Concrete B, and Concrete C) and one generic (Generic) (see Table 1). The Concrete A instantiation was used in prior research and was shown to facilitate quick learning of the rules of the mathematical group (1). The symbols of this instantiation were three images of measuring cups containing varying levels of liquid (see Table 1). Participants were told they need to determine a remaining amount when different measuring




cups of liquid are combined. In particular,  and  will fill a container. So for example, combining  and  would fill one container and have  remaining. Additionally, participants were told that they should always report a remainder. Therefore they should report that the combination of  and  will have remainder . For this instantiation, the elements are familiar with known uses; and the storyline most likely taps participants' prior knowledge of containers, quantities, and pouring which could help to convey the to-be-learned principles of the group structure. In this case, the storyline and symbols facilitate learning.

Concrete B and Concrete C instantiations were constructed similarly with storylines and symbols that would assist learning. The task for the Concrete B instantiation was to determine the amount of burned pizza served from a restaurant where the cook systematically burned a portion of every order. Three possible individual orders could be placed: 1, 2, or 3 slices, represented as







, , and . The proportion that would be burned followed the rules of the mathematical group. For example, when an order for  and  was placed, then  would be burned. For the Concrete C instantiation, participants were told of a tennis ball manufacturing machine.

Instead of producing batches of 3 balls to fill each ball container, the machine was producing batches of 0, 1, or 2 balls, represented as , , and . Under these circumstances, more than one batch of balls would be combined to fill containers. Participants' task was to determine the extra balls resulting from combining batches. For example, if these two batches were combined,  and , then  would be extra.

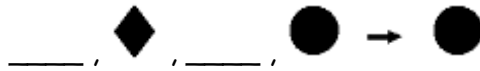
The Generic instantiation was presented as a symbolic language in which three types of symbols combine to yield a resulting symbol. Combinations were expressed as written statements of the form: *symbol 1, symbol 2* \rightarrow *resulting symbol*. For example,

,  \rightarrow . In this case, the instantiation is generic in the sense that the symbols are not necessarily meaningful, the storyline is novel, and the rules of the language are arbitrary.

In the Concrete 1 condition, participants learned the Concrete A instantiation. In the Concrete 2 condition, participants learned Concrete A and Concrete B instantiations. In the Concrete 3 Condition, they learned Concrete A, B, and C instantiations. Participants in the Generic 1 condition learned the Generic instantiation. In the conditions in which participants learned more than one instantiation, they were explicitly told that the instantiations followed similar rules. Specifically, they were told after learning an instantiation, “Now you will learn about a new system. This system works the same ways as the last system you learned. The rules of the last system are like the rules of this new one.” Training was equated across condition, so that the same examples, questions with feedback, summaries of rules, and test questions were spread across the learning instantiations. For each instantiation, training consisted of an introduction and explicit presentation of the rules through examples. For instance, for Concrete A, participants

were told that combining  and  has a remainder of . Analogously, for the Generic instantiation where students were told that symbols combine to yield a resulting symbol, the analogue to the above rule was presented as ,  \rightarrow . Questions with feedback were given; and complex examples were shown. To approximate the effect of presenting the rules in more than one domain, additional summaries of the rules were given when learning fewer domains. After training of an instantiation, the participants were given a multiple-choice test designed to measure the ability to apply the learned rules to novel problems. In total, 24 multiple-choice questions were posed in phase 1 (learning phase). Questions were distributed evenly over the learning domains. Participants who learned two instantiations had a 12-question test over the first domain and the remaining twelve questions over the second domain. Participants who learned three instantiations had 8-question tests over each domain. Participants who learned only one instantiation were given a 24-question test. Many questions required application of multiple rules. The following are examples of test questions for the Generic instantiation.

(1) What can go in the blanks to make a correct statement?



(2) Find the resulting symbol:



The concrete instantiations presented the analogues of these questions. Appendix A presents excerpts from training and all test questions for the different instantiations.

After training and testing of the learning domain(s), a novel transfer domain was presented. The same transfer domain was used for all four conditions and was described as a children’s game involving three objects (see Table 1). Children sequentially point to objects and a child who is “the winner” points to a final object. The correct final object is specified by the rules of the game (rules of a mathematical group). Participants received no explicit training in the target domain. They were not explicitly taught the rules of the system. Instead they were told that the game rules were like the rules of the system(s) they just learned and they need to figure them out by using their newly acquired knowledge (i.e. transfer). After being asked to study a series of

examples from which the rules could be deduced, a 24-question multiple-choice test was given. The questions on this test were isomorphic to those given in phase 1. Questions were presented individually on the computer screen along with four key examples at the bottom of the screen. The same four examples were shown with all test questions. Excerpts from the presentation of the target instantiation and all test questions are presented in Appendix B.

Procedure Training and testing were presented to individual participants on a computer screen in a quiet room. They proceeded through training and testing at their own pace; and their responses were recorded.

Results Eight participants (three Concrete 2, three Concrete 3, and two Generic 1) were eliminated from the analysis for failing to learn as evidenced by learning score(s) not above chance. Chance score for composite learning (the sum of scores in all learning instantiations) was 9 out of 24, or 37.5%. The criterion for learning was composite learning score as well as learning scores in each individual instantiation above chance. In all conditions, participants successfully learned the material see Table 2. Both composite learning scores and scores in each individual learning domain were above chance, one-sample t-tests, $t_s > 9.62$, $p_s < .001$, $t_s > 8.84$, $p_s < .001$, respectively. One-way ANOVA was performed on composite learning scores. No differences were found across condition, $F(3, 68) = .697$, $p > .55$.

Transfer scores differed across condition ($M = 44\%$, $SD = 16.0\%$ Concrete 1, $M = 44\%$, $SD = 17.2\%$ Concrete 2, $M = 51\%$, $SD = 20.3\%$ Concrete 3, $M = 76\%$, $SD = 21.6\%$ Generic 1), one-way analysis of variance, $F(3, 68) = 11.93$, $p < .001$, $\eta^2 = .345$. Participants in the Generic 1 condition performed higher than those in the other conditions, post-hoc Tukey comparisons $p_s < .002$. There were no differences across the concrete conditions, post-hoc Tukeys, $p_s > .626$. Transfer in the Generic 1 condition was above chance (chance score was 9 out of 24, or 37.5%), one-sample t-test, $t(17) = 7.55$, Bonferroni adjusted $p < .005$. Transfer in Concrete 1 and Concrete 2 conditions did not differ from chance, $t_s < 1.7$, Bonferroni adjusted $p_s > .35$. Transfer in Concrete 3 condition was marginally above chance, $t(16) = 2.78$, $p = .052$, Bonferroni adjusted. Across condition, participants spend approximately 25 minutes on the experiment, with no differences in time across condition, Analysis of Covariance with learning and transfer scores as covariates, $F(3, 68) = 1.427$, $p = .243$, $\eta^2 = .061$.

In previous experiments (1), no participant was able to score above chance on a test of the transfer domain without first learning an isomorphic domain. Therefore, in the present experiment, transfer test scores that are above chance suggest successful transfer of conceptual knowledge.







Also, two separate experiments were conducted to control for similarity of domains. In the first, undergraduate students read descriptions of the learning and transfer domains, but received no explicit training of the rules. Participants found both the generic and concrete domains equally similar to the target domain, independent sample $t(38) = .717$, $p = .478$. In the second experiment, undergraduate participants were assigned to two learning conditions, Generic-Concrete or Concrete-Generic. The Generic-Concrete condition was identical to the Generic 1 condition of Experiment 1; participants learned the Generic instantiation and were presented with the transfer task. The Concrete-Generic condition reversed the learning and transfer instantiations. In this condition, participants first were explicitly taught the to-be-learned mathematical rules using the Concrete domain that was the transfer domain of Experiment 1 (i.e. the children's game). Then they were presented with a transfer task involving the Generic instantiation. As in Experiment 1, in the transfer phase, participants were told to apply their knowledge of the first system they learned to figure out the rules of the second in which they saw only examples prior to being tested. The results were that participants in the Generic-Concrete condition had significantly higher transfer scores than those in the Concrete-Generic condition, independent samples t-test $t(24) > 3.5$, $p < .003$. This asymmetry in transfer cannot stem from differential similarity because the two learning conditions

used the same pair of instantiations. Taken together, the results of these two control experiments suggest that any differences in transfer performance across conditions cannot be attributed to differential similarity of learning and transfer domains.

Experiment 2

Participants Twenty undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course.

Materials, Design and Procedure The materials and procedure of this experiment were almost identical to that of Experiment 1, with two differences. First, all participants learned two concrete instantiations, Concrete A and Concrete B. Second, participants were given the correspondence of analogous elements between the two learning instantiations. Therefore, participants were trained on Concrete A instantiation and were given a 12-question multiple-choice test. Then participants

were trained on Concrete B instantiation and told that  is like  ;  is like  ;  is like  . Participants were given a 12-question multiple-choice test on Concrete B instantiation. As in Experiment 1, participants were presented with the novel transfer domain, shown examples, and given a 24-question multiple-choice test analogous to the questions posed over the two concrete learning instantiations.

Results All participants successfully learned. Scores in each learning domain were above chance ($M = 88\%$, $SD = 9.6\%$ and $M = 77\%$, $SD = 16.2\%$, Concrete A and B respectively), one sample t-test, $t(19)s > 12.0$, $ps < .001$. Transfer scores were not above chance ($M = 41\%$, $SD = 16.7\%$), one-sample t-test $t(19) = .943$, $p > .356$.

Experiment 3

Participants Twenty undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course.

Materials, Design and Procedure The materials and procedure of this experiment were very similar to that of Experiment 2. All participants learned two concrete instantiations, Concrete A and Concrete B. Unlike Experiment 2, participants were not given the correspondence of analogous elements between the two learning instantiations. Instead after learning Concrete A and Concrete B instantiations, participants were asked to write on paper any similarities they observed between the two learning instantiations. They were also asked to match analogous elements. Afterward, they proceeded to the transfer phase which was identical to that of Experiments 1 and 2.

Results Two participants were removed from the analysis for failing to learn the material; scores in one of the learning domains were no different than a chance score of 5 out of 12. Participants successfully learned the material. Both composite learning scores ($M = 88.0$, $SD = 11.0$) and scores in each individual learning domain ($M = 90.3$, $SD = 11.9$ for Concrete A; $M = 85.6$, $SD = 13.0$ for Concrete B) were above chance, one-sample t-tests, $t(17) > 19.52$, $p < .001$, $t(17)s > 14.31$, $ps < .001$, respectively.

The distribution of transfer scores was bimodal. Approximately 44% of participants scored highly on the transfer test (scores of 88%-100%, $M = 95\%$, $SD = 4.7\%$). However, the remaining participants did not transfer well, (scores of 33%-70%, $M = 51\%$, $SD = 11.6\%$). An independent sample t-test was performed to consider difference in learning scores between participants who

transferred and those who did not ($M = 93\%$, $SD = 4.4\%$; $M = 84\%$, $SD = 12.9\%$, respectively), $t(16) = 1.97$, $p = .066$, suggesting that the process of comparison helped higher learners, but not others.

Experiment 4

Participants Forty undergraduate students from Ohio State University participated in the experiment and received partial credit for an introductory psychology course.

Materials, Design and Procedure The materials, and procedure were similar to those of the previous experiments. There were two between-subject conditions: Generic and Concrete-then-Generic. In the Generic condition, participants learned only the generic instantiation in phase 1. In the Concrete-then-Generic condition, participants learned the Concrete A and then the Generic instantiation. As in the previous experiments, training was equated across condition and there were 12-question multiple-choice tests in each of the two learning domains. Appendix A indicates the specific questions that were tested in each instantiation. After the learning phase, all participants were presented with the transfer instantiation and the 24-question multiple-choice test as in the previous experiments.

Results Seven participants (one Generic and six Concrete-then-Generic) were removed from the analysis for failing to learn the material; scores in one of the learning domains were no different than a chance score of 5 out of 12 or 9 out of 24 for the Generic condition. Participants successfully learned the material. In the Concrete-then-Generic condition, both composite learning scores ($M = 83.9$, $SD = 10.6$) and scores in each individual learning domain ($M = 88.1$, $SD = 8.33$ for Concrete A; $M = 79.8$, $SD = 17.5$ for Generic) were above chance, one-sample t-tests, $t(13) > 16.4$, $p < .001$, $t(13)s > 8.13$, $ps < .001$, respectively. Learning scores in the Generic condition ($M = 81.4$, $SD = 9.36$) were also above chance, one-sample t-test, $t(18) > 20.4$, $p < .001$. There was no difference in composite learning scores across condition, independent samples t-test, $t(31) = 1.42$, $p = .466$. There were also no differences in times ($M = 29.4$ seconds, $SD = 6.9$ for the Concrete-then-Generic; $M = 27.6$ seconds, $SD = 5.7$ for the Generic condition), independent samples t-test, $t(31) = .808$, $p > .42$.

Participants in both conditions, successfully transferred ($M = 65.5$, $SD = 26.2$ in the Concrete-then-Generic; $M = 83.3$, $SD = 10.6$ in the Generic), transfer scores were above chance, one sample t-tests, $t(13) > 3.99$, $p < .003$ and $t(18) > 18.8$, $p < .001$ respectively. However, participants in the Generic condition performed markedly higher than those in the Concrete-then-Generic condition, independent samples t-test, $t(31) > 2.69$, $p < .012$.

Table 1: Stimuli and rules across Generic, Concrete A, and Transfer instantiations.

Rules of Commutative Group: with operation denoted as +						
Elements	For a group of order 3, there are 3 unique elements: x, y, z					
Associative	For any elements x, y, z : $((x + y) + z) = (x + (y + z))$					
Commutative	For any elements x, y : $x + y = y + x$					
Identity	There is an element, I , such that for any element, x : $x + \mathbf{I} = x$					
Inverses	For any element, x , there exists another element, y , such that $x + y = \mathbf{I}$					
The Specific Instantiations of a Group:						
	Generic		Concrete A		Transfer Domain	
<u>Elements</u>						
<u>Specific Rules:</u>	is the identity		is the identity		is the identity	
	Operands	Result	Operands	Result (Remainder)	Operands	Result

Table 2: Learning scores (% correct) by instantiation. Note chance = 37.5%.

	1 st Instant.	2 nd Instant.	3 rd Instant.	Composite Learning Score
Condition	<i>Mean (SD)</i>	<i>Mean (SD)</i>	<i>Mean (SD)</i>	<i>Mean (SD)</i>
Concrete 1	75.8 (17.8)	-	-	75.8 (17.8)
Concrete 2	82.3 (15.6)	74.5 (19.2)	-	78.4 (16.5)
Concrete 3	80.1 (16.0)	78.6 (13.8)	89.0 (12.4)	82.6 (8.75)
Generic 1	80.3 (13.7)	-	-	80.3 (13.7)

References

1. Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2005). Relevant concreteness and its effects on learning and transfer. In B. Bara, L. Barsalou & M. Bucciarelli (Eds.), *Proceedings of the XXVII Annual Conference of the Cognitive Science Society*.

Appendix A

Learning Instantiations

Excerpt from training of the Generic Instantiation:

On an archaeological expedition, tablets were found with inscriptions of statements in a symbolic language. The statements involve these three symbols: ● ◆ ◀ and follow specific rules.

Rules for combining symbols.

Rule 1. The order of the two symbols on the left does not change the result.

For example $\blacklozenge, \blacktriangleleft \rightarrow \blacklozenge$
 is the same thing as $\blacktriangleleft, \blacklozenge \rightarrow \blacklozenge$

Rule 2. When any symbol combines with ◀, the result will always be the other symbol.

For example:
 $\blacktriangleleft, \blacklozenge \rightarrow \blacklozenge$ and
 $\bullet, \blacktriangleleft \rightarrow \bullet$

Rule 3. $\bullet, \blacklozenge \rightarrow \blacktriangleleft$

Rule 4. $\bullet, \bullet \rightarrow \blacklozenge$

Rule 5. $\blacklozenge, \blacklozenge \rightarrow \bullet$

Rule 6. The result does not depend on which two symbols combine first.

For example: $\blacklozenge, \blacktriangleleft, \bullet \rightarrow \blacktriangleleft$
 It does not matter if we do
 $\blacklozenge, \blacktriangleleft$ first and then \bullet or $\blacktriangleleft, \bullet$ first and then \blacklozenge .

Test Questions for Generic Instantiation:

1. Find the resulting symbol: $\bullet, \blacktriangleleft \rightarrow \underline{\hspace{2cm}}$









Choose: 1.) ◀ 2.) ◆ 3.) ●

2. Find the resulting symbol: 




- Choose: 1.)  2.)  3.) 









3. What symbols go in the blanks to make a correct statement?

 ,  ,  → 




- Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 

4. What goes in the blanks to make a correct statement?

 ,  → 

- Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 

5. What goes in the blank to make a correct statement?

 ,  → 

- Choose: 1.)  2.)  3.) 

6. What can go in the blank to make a correct statement?


 ,  → 

- Choose: 1.)  ,  2.)  , 
 3.)  ,  4.)  , 

7. What expression has the same result as the following expression?




 ,  ,  ,  → 

- Choose: 1.)  ,  2.)  , 

- 3.)  4.) none of the above

8. Some of my team members were discussing what symbol could be placed in the first blank below. Which of their responses do you agree with?



- Choose: 1.) any symbol 2.) any symbol except 
 3.) any symbol except  4.) any symbol except 

9. When we were analyzing tablets, I overheard two of my team members talking. They were arguing about whether these inscriptions mean the same thing (have the same result). What do you think?



- Choose: 1.) same 2.) different

10. How about the following? Do they mean the same thing?



- Choose: 1.) same 2.) different









11. Do the following give the same result?









- Choose: 1.) same 2.) different



12. What goes in the blanks to make a correct statement?



- Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 

13. Which of the following symbols combine to give  ?

- Choose:
- 1.)  and 
 - 2.)  and 
 - 3.)  and 
 - 4.) none of the above

14. How many  's could combine with themselves to get  ?

- Choose:
- 1.) four
 - 2.) five
 - 3.) six
 - 4.) seven

15. Find the resulting symbol:







- Choose:
- 1.) 
 - 2.) 
 - 3.) 

16. What symbol goes in the blank to make a correct statement?



- Choose:
- 1.) 
 - 2.) 
 - 3.) 

17. When we were working, a tablet was broken. We tried to figure out what it stated. We did not know the result, but we did know that there were two symbols on the left; one of them was . We were trying to figure out what the result could be. Here are some opinions of my team members. Which do you agree with?

- Choose:
- 1.) the result could be any symbol
 - 2.) the result could only be  or 
 - 3.) the result can only be 

18. Find the resulting symbol:



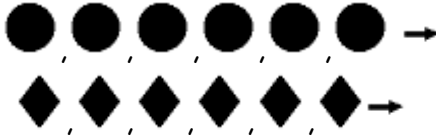
- Choose:
- 1.) 
 - 2.) 
 - 3.) 

19. Do the following statements mean the same thing?



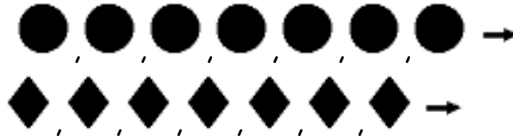
Choose: 1.) same 2.) different

20. Do the following statements mean the same thing?



Choose: 1.) same 2.) different

21. Do the following statements mean the same thing?



Choose: 1.) same 2.) different

22. What goes in the blank to make a correct statement?



Choose: 1.)  2.)  3.) 

23. What is the result of the following?



Choose: 1.)  2.)  3.) 

24. What goes in the blank to make a correct statement?






Choose: 1.)  ,  2.)  , 
 3.)  ,  4.) none of the above







In Concrete-then-Generic condition of Experiment 4:




These questions were tested: 2, 4, 6, 8, 12, 13, 14, 16, 19, 20, 21, 23


Excerpt from training of the Concrete A Instantiation:







A company makes detergents by mixing three different quantities of solutions, represented as , , and . The company is testing the mixtures and wants to know what amount of solution is left-over in the mixing process.




Rules for finding left-over quantities:

Rule 1. The order by which two cups of solution are combined does not change the left-over result. For example, combining  with  has a left-over quantity of . And combining  with  has the left-over quantity .

Rule 2.  and  will fill a container, but we need a quantity of solution to test, so we consider  as the left-over.





Rule 3. When any kind of cup of solution combines with , the result will always be the other solution cup. For example:







When  and  combine,  is left-over.
And when  and  combine,  is left-over.

Rule 4. A combination of  and  does not fill a container, so the left-over is .



Rule 5. A combination of  and  fills one container and has  left-over.




Rule 6. Finally, you need to know that when mixing more than 2 cups of solution, the order of combining solutions does not matter. The left-over is the same no matter which cups are combined first. For example:

When we combine ,  and , the left-over is .

It does not matter if we do  and  first and then 
OR  and  first and then .




Test Questions for Concrete A Instantiation:



1. What is left-over when  and  combine?









Choose: 1.)  2.)  3.) 


2. What is left-over when the following cups of solution are combined?











Choose: 1.)  2.)  3.) 




3. What possible cups of solution can combine with  to have a left-over of  ?



Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 


4. Which cups of solution can combine to have  left-over?

Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 

5. What can combine with  to have  left-over?







Choose: 1.)  2.)  3.) 




6. What can mix with  to have  as a left-over?




Choose: 1.)   2.)  
 3.)   4.)  

7. What combination of cups has the same left-over as the following?




Choose: 1.)   2.)  
 3.)   4.) none of the above

8. Some of the Bubblinski employees were analyzing a batch of detergent. The left-over was . There were 4 cups of solution that were mixed. Two of these cups were  and , but the other two cups were not known. The guys were discussing which other cups could possibly have been involved in the mix. Which of their responses do you agree with?

- Choose:
- 1.) any cup
 - 2.) any cup except 
 - 3.) any cup except 
 - 4.) any cup except 

9. Later I overheard two employees talking. They were arguing about whether these two mixtures of solution (below) would have the same left-over. What do you think?


Mix 1: 


Mix 2: 

- Choose:
- 1.) same
 - 2.) different

10. True or false...


When the cups are mixed, mix 1 and mix 2 will have the same left-over.


Mix 1: 

Mix 2: 




- Choose:
- 1.) true
 - 2.) false









11. How about these mixtures, will they have the same left-overs?


Mix 1: 







Mix 2: 



- Choose:
- 1.) true
 - 2.) false

12. What cups can combine with  and  to result in a left-over of  ?

- Choose:
- 1.)  and 
 - 2.)  and 
 - 3.)  and 
 - 4.)  and 

13. Which cups can combine to give a left-over of  ?

- Choose:
- 1.)  and 
 - 2.)  and 
 - 3.)  and 
 - 4.) none of the above

14. How many  's could combine with themselves to get  ?




- Choose:
- 1.) four
 - 2.) five
 - 3.) six
 - 4.) seven

15. What is left-over when the following cups are mixed?







- Choose:
- 1.) 
 - 2.) 
 - 3.) 

16. What cup can mix with the following and have  left-over?

- Choose:
- 1.) 
 - 2.) 
 - 3.) 
 - 4.) we need more information to answer

17. One day, a batch of detergent was spilled. We did not know the left-over quantity, but we

did know that there were two cups in the mixture; one of them was  . We were trying to figure out what the left-over could have been. Here are some opinions of the employees. Which do you agree with?

- Choose:
- 1.) the left-over could be any cup
 - 2.) the left-over could only be  or 
 - 3.) the left-over could only be 

18. What is left-over when the following cups of solution combine?



- Choose:
- 1.) 
 - 2.) 
 - 3.) 

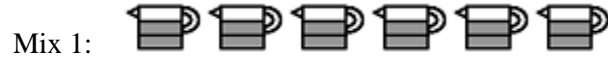
19. Do the following mixtures have the same left-overs?





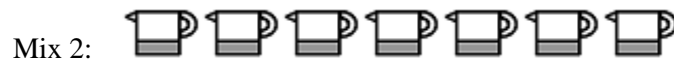
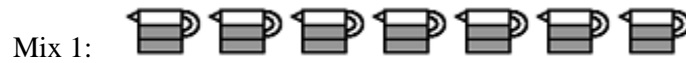
Choose: 1.) yes 2.) no

20. Do the following mixtures have the same left-overs?



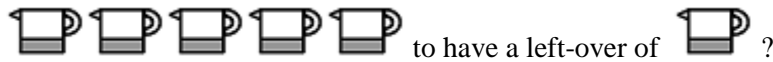
Choose: 1.) yes 2.) no

21. How about the following, do they have the same left-over?



Choose: 1.) yes 2.) no

22. What cup needs to mix with the following



Choose: 1.) 2.) 3.)

23. What is left-over when the cups below are mixed?



Choose: 1.) 2.) 3.)

24. What cups need to mix with to have left-over?

Choose: 1.) 2.) 3.) 4.) none of the above







In Concrete 2 condition of Experiment 1, Experiment 3, and Concrete-Generic condition of Experiment 4:

These questions were tested: 1, 3, 5, 7, 9, 10, 11, 15, 17, 18, 22, 24







In condition Concrete 3:








These questions were tested: 1, 3, 6, 9, 10, 15, 22, 24




Excerpt from training of the Concrete Instantiation B:




A pizzeria takes orders for one, two or three slices represented on individual order cards as , , and . Multiple orders are placed at a time; and the cook systematically burns a portion of each group order. Antonio needs help to determine how much pizza is burned. There is never more than 1 whole pizza burned. So the burned amount will always be , , or .




Rules for finding how much pizza is burned stated by Antonio:

Rule 1. What I order first or second doesn't matter. The same amount gets burned. For example, if I order this  first and then this , then this much  is given to us burned. The same thing happens if I order this  first and then this . We get this much  burned.





Rule 2. If I order this  with any other single amount, the other amount is always burned. Here are a couple of examples: If I order  and , then  is burned. If I order  and , then  is burned.




Rule 3. If I order  and , then only one pizza is made and  is burned.

Rule 4. If I order  and , then one pizza is made and  is burned.

Rule 5. If I order  and , then one pizza is ok, but  is burned.

Rule 6. If I turn in more than 2 order cards, the order that I turn them in doesn't matter. The same amount ends up burned. For example,


if I turn in  and  and then , then  is burned.









The same amount is burned if I turn in  and  and then .



Test Questions for Concrete Instantiation B for Concrete 2 condition:









1. How much is burned in the following order?   




Choose: 1.)  2.)  3.) 




2. What order could have been placed so that this much  was burned?

Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 









3. What can be ordered with  to have  burned?


Choose: 1.)   2.)  
 3.)   4.)  







4. The other day, some coworkers were arguing about the pizza order that was placed. This much  was burned. And they knew that these cards   were part of the order. The guys were discussing what other cards could have been part of the order. Which of their responses do you agree with?



Choose: 1.) any card 2.) any card except 
 3.) any card except  4.) any card except 

5. What could have been ordered with   to get this much  burned?


Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 

6. What could be ordered to have this much  burned?

Choose: 1.)  and  2.)  and 
 3.)  and  4.) none of the above

7. How many of these  could be ordered to have this much  burned?

Choose: 1.) four 2.) five
 3.) six 4.) seven

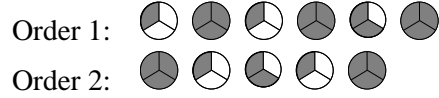
8. What additional amount can be ordered with the following to get this  burned?



Choose:

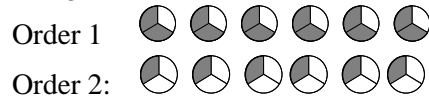
- 1.)  2.) 
 3.)  4.) we need more information to answer

9. Do the following orders have the same burned amount?



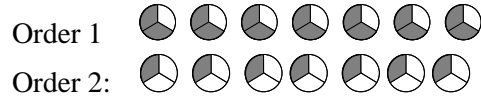
Choose: 1.) Yes 2.) No

10. Do the following orders have the same burned amount?



Choose: 1.) Yes 2.) No

11. Do the following orders have the same burned amount?



Choose: 1.) Yes 2.) No

12. How much will be burned in this order?









Choose: 1.)  2.)  3.) 

In condition Concrete 3:




These questions were tested: 1, 2, 4, 8, 10, 11, 12, and also the following question:

What order has the same burned amount as the following order?
















Choose: 1.)   2.)  
 3.)   4.) none of the above




Excerpt from training of the Concrete Instantiation C:




A tennis ball manufacturing company is having trouble with their ball-making machine. Instead of producing batches of three balls to fill a container, it is producing batches of zero, one or two balls represented as , , and . Consequently two or more batches need to be produced to fill a container. In doing so, the number of extra balls produced needs to be determined.




Rules for finding the number of extra tennis balls:

Rule 1. The order of the batches doesn't matter. The number of extra balls will be the same. For example, if this batch  is made first and then this , then this much  is extra. The same thing happens if this batch  is made first and then this . We will have this much  extra.

Rule 2. If this batch  is made with any other single batch, the other amount is always extra. Here are a couple of examples: If this  and this  are made, then  is extra. If this  and this  are made, then  is extra.




Rule 3. If  and  are produced, then one container can be filled and  is extra.

Rule 4. If  and  are produced, then we cannot fill a container. So,  is extra.



Rule 5. If  and  are made, then one container can be filled and  is extra.




Rule 6. If more than two batches are produced, the order in which they are made doesn't matter. The extra will be the same. For example,

if  and  and then  are made, then  is extra. The same amount is extra










if  and  and then .










Test Questions for Concrete Instantiation C for Concrete 3 condition:

1. Two batches of balls were made and  was extra. One batch was . What was the other batch?




Choose: 1.)  2.)  3.) 









2. Two groups of batches were made. Do they have the same amount extra?


Group 1         




Group 2         



Choose: 1.) Yes 2.) No

3. Four batches of balls were made including  and . This  was extra. What were the other two batches?


Choose: 1.)  and  2.)  and 
 3.)  and  4.)  and 




4. Which two batches below could have been made so that  is extra?

Choose: 1.)  and  2.)  and 
 3.)  and  4.) none of the above




5. How many batches of  could be made so that  is extra?

Choose: 1.) four 2.) five
 3.) six 4.) seven







6. One day two batches of balls were made. One of the batches was  , but we didn't know the other. Some guys at the factory were arguing about how much would be extra. Which do you agree with?




Choose: 1.) the extra could be any amount
 2.) the extra could only be  or 
 3.) the extra can only be 

7. How much is extra if these batches were produced?   

Choose: 1.)  2.)  3.) 

8. Two groups of batches were made. Do they have the same the same amount extra?

Group 1      

Group 2     

Choose: 1.) Yes 2.) No

Appendix B
Test of Transfer Domain

Excerpt from the Introduction:

In another country, children play a pointing game that involves these three objects:



. Children point to objects and the winner points to the correct final object. The rules of the last system you learned are like the rules of this game. So use what you know about the last system to help you figure out the rules of this game.

Test Questions for Transfer Domain:

The examples in the table below were presented with each of the following test questions.

If the kids point to these:				
Then the winner points to this:				

1. What object do you think the winner will point to when the other kids point to then ?

Choose: 1.) 2.) 3.)


2. What object does the winner point to when the other kids point to then then ?

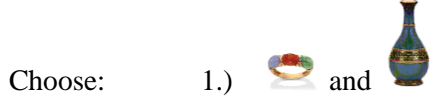
Choose: 1.) 2.) 3.)



3. If a group of kids wants the winner to point to , and they first point to , what other objects do they need to point to?

Choose: 1.) and 2.) and





4. If the winner pointed to  , what objects might the other kids have pointed to?







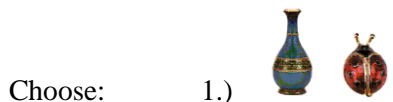
5. What object do the children need to point to along with  , so that the winner points to  ?






6. What objects can the kids point to along with  so that the winner points to  ?



7. Suppose that a group kids points to these objects:     . If other groups (in different games) point to the objects below, which group would have a winner pointing to the same object as the group above?






4.) none of the above

8. A group of kids wants the winner to point to  . They have already pointed to  and  ; and they want to point to two more. What can their next object be?

3.)  and 

4.)  and 



13. What objects should the kids point to so that the winner points to  ?

Choose: 1.)  and 

2.)  and 

3.)  and 

4.) none of the above

14. How many times could the kids point to  so that the winner points to  ?

Choose: 1.) four

2.) five

3.) six

4.) seven

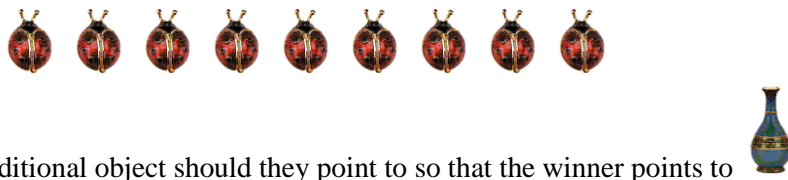
15. After the children point to these objects:







What will the winner point to?


Choose: 1.)  2.)  3.) 

16. The children point to these objects:






What additional object should they point to so that the winner points to  ?

Choose: 1.)  2.)  3.)  4.) we need more information to answer

17. Three children were playing together. One child pointed to  . They were going to point to one more object, but before they did, they were trying to decide which object the winner would point to. Here are their opinions about the winning object. Which do you agree with?

Choose: 1.) the left-over could be any cup

2.) the left-over could only be  or 

3.) the left-over could only be 

18. If the kids point to these objects:
What object will the winner point to?



Choose: 1.)  2.)  3.) 

19. In separate games, one group of kids pointed to these objects:



And another group pointed to these:



Will the winners of each game point to the same object?

Choose: 1.) yes 2.) no

20. How about these objects?
In one game, the kids point to these objects:



and in another game, the kids point to these:



Will the winners of each game point to the same object?

Choose: 1.) yes 2.) no

21. How about these objects?
In one game, the kids point to these objects:



and in another game, the kids point to these:




Will the winners of each game point to the same object?

Choose: 1.) yes 2.) no

22. A group of children pointed to these objects:



What additional object do they need to point to so that the winner will point to  ?

Choose: 1.)  2.)  3.) 


23. What will the winner point to if the children point to these?



Choose: 1.)  2.)  3.) 

24. If the group of children point to these objects:



What additional objects should they point to so that the winner will point to  ?

Choose: 1.)   2.)  
 3.)   4.) none of the above