www.sciencemag.org/cgi/content/full/318/5856/1601/DC1

Supporting Online Material for

Economics of Overexploitation Revisited

R. Q. Grafton,* T. Kompas, R. W. Hilborn

*To whom correspondence should be addressed. E-mail: quentin.grafton@anu.edu.au

Published 7 December 2007, *Science* **318**, 1601 (2007)
DOI: 10.1126/science.1146017

This PDF file includes:

Materials and Methods
References

Supplementary Material for ‘Economics of Overexploitation Revisited’

To calculate MEY we must specify and derive the harvest, fishing effort, revenue, cost and profit functions and the population dynamics that form the basis of the stochastic dynamic optimization. In a multi-species, multi-fleet model, effort allocation, or the allocation of harvest across vessels and species must also be specified.

Harvest function

The harvest function of vessel type *j* for species *i* at time *t* is given by

$$h_{jit} = q_{ji}^{0} E_{jit}^{\alpha_{ji}} B_{it}^{\beta_{ji}} \tag{1}$$

where q_{ji}^{0} is a catchability rate of vessel type *j* to species *i*; E_{jit} is the fishing effort of vessel type *j* to species *i* at time *t*; B_{it} is the biomass stock of species *i* at time *t*; and α_{ji} and β_{ji} are the parameters in the harvest function of vessel type *j* targeting species *i* (assumed to be constant over time).

Fishing effort allocation

From (1) the fishing effort of vessel type *j* to fish species *i* is given as

$$E_{jit} = \left(\frac{h_{jit}}{q_{ji}^{0} B_{it}^{\beta_{ji}}} \right)^{1/\alpha_{ji}} \tag{2}$$

and

$$h_{it} = \sum_{j=1}^{n} h_{jit} = h_{it} \tag{3}$$

and

$$E_{jt} = \sum_{i=1}^{m} E_{jit} \tag{4}$$

where h_{it} is the total allowable catch (TAC) for species *i* at time *t* and E_{jt} is the fishing effort of vessel type *j* at time t. Denote the harvest share of vessel type *j* for species *i* at time *t* given a TAC of that species as θ_{ijt}. Thus the harvest h_{jit} can be expressed as

$$h_{jit} = \theta_{jit}\, h_{it} \tag{5}$$

where the shares must sum to one, or $\sum_{j=1}^{n} \theta_{jit} = 1$.

Substitution of (5) into (1) gives

$$E_{jit} = \left(\frac{1}{q_{ji}^{0}} \frac{h_{it}\, \theta_{jit}}{B_{it}^{\beta_{ji}}} \right)^{1/\alpha_{ji}} \tag{6}$$

that defines the relationship between biomass and TAC on the effort allocated to species *i*.

Revenue function

Annual total revenue of vessel type *j* at time *t* (TR_{jt}) is defined as a sum of revenue of all targeted species landed by that vessel, which is calculated as the multiple of harvest and the annual (average) price of each species of fish, or

$$TR_{jt} = \sum_{i=1}^{m} TR_{jit} = \sum_{i=1}^{m} h_{ijt}\, p_{it} \tag{7}$$

where p_{it} is the price of species *i* at time *t*. The price of fish (p_{it}) is itself determined from an inverse demand curve as follows

$$p_{it} = p_i^0 (H_i^0 / \sum_{j=1}^{n} h_{ijt})^{1/\varepsilon_i} \tag{8}$$

where ε_i is the elasticity of demand for catch for species *i* and p_i^0 is the unit price of catch when the volume of the catch is H_i^0. Total revenue of the fishery at time *t* (TR_t) is defined as a sum of revenue of all vessel types at that time, or

$$TR_t = \sum_{j=1}^{n} \sum_{i=1}^{m} p_{it} h_{ijt} \tag{9}$$

Fishing cost function

Fishing costs (including labor, material, capital and all other costs) are defined as a function of fishing effort and biomass. Fishing costs for vessel type *j* for species *i* at time *t* (c_{jit}) depend on a fixed cost component and variable costs which depend on the fishing effort of vessel type *j* on species *i* (E_{jit}), i.e.,

$$c_{jit} = \gamma_{jt}^0 + \gamma_{jt}^1 E_{jit} \tag{10}$$

where γ_{jt}^0 is the fixed cost parameter of vessel type *j* and γ_{jt}^1 is the variable cost share parameter and both are positive.

Substitution of (6) into (10) yields

$$c_{jit} = \gamma_{jt}^0 + \gamma_{jt}^1 \left(\frac{h_{it}\, \theta_{jit}}{q_{ji}^0 B_{it}^{\beta_{ji}}} \right)^{1/\alpha_{ji}} \tag{11}$$

where the smaller is the biomass the larger is the cost of fishing. Total fishing costs of vessel type *j* (c_{jt}) are given by

$$c_{jt} = \sum_{i=1}^{m} c_{jit} \tag{12}$$

The total fishing cost for the fishery as a whole is a sum of total fishing costs of all vessel types in the fishery.

Profit function

Annual economic profit of vessel type *j* for species *i* at time *t* (Π_{jit}) is defined by subtracting annual total cost from annual total revenue, i.e.,

$$\Pi_{jit} = p_{it} h_{it} \theta_{jit} - \left[\gamma_{jt}^{0} + \gamma_{jt}^{1} \left(\frac{h_{it}\,\theta_{jit}}{q_{ji}^{0} B_{it}^{\beta_{ji}}} \right)^{1/\alpha_{ji}} \right] \tag{13}$$

and total profit in the fishery across vessels and species at time *t* (Π_t) is given by

$$\Pi_t = \sum_{i=1}^{m} p_{it}\, h_{it} - \sum_{j=1}^{n} \sum_{i=1}^{m} \left[\gamma_{jt}^{0} + \gamma_{jt}^{1} \left(\frac{h_{it}\,\theta_{jit}}{q_{ji}^{0} B_{it}^{\beta_{ji}}} \right)^{1/\alpha_{ji}} \right]. \tag{14}$$

Objective function

The optimization problem is to maximize the aggregate profit over a period of time *T* through choice of the harvest (TAC) for each species and the harvest share allocated among vessel types. In other words, the problem is to

$$\max_{h_{it},\theta_{ijt}} \Pi_t = \sum_{t=0}^{T} \left(\frac{1}{(1+\delta)^t} \right) \left\{ \sum_{i=1}^{m} p_{it}\, h_{it} - \sum_{j=1}^{n} \sum_{i=1}^{m} \left[\gamma_{jt}^{0} + \gamma_{jt}^{1} \left(\frac{h_{it}\, \theta_{jit}}{q_{ji}^{0} B_{it}^{\beta_{ji}}} \right)^{1/\alpha_{ji}} \right] \right\} \tag{15}$$

where δ is the discount rate. Solving equation (15) also requires that the unexploited biomass at time 0 for each species is known.

Population Dynamics

The biomass in the northern prawn fishery is modelled using a Ricker spawner-recruitment model described in detail in Quinn and Deriso (1999, pp. 89-92). The biomass in the Australian orange roughy fishery and the western and central Pacific yellowfin and bigeye tuna fisheries is described by a Beverton-Holt (1957) age structured model, with a specification for a length-weight relationship as follows. The stock recruitment relationship is given by

$$R_t = \frac{\mu_3 B_{t-1}}{1 + \dfrac{\mu_4 B_{t-1}}{B_0}} \tag{16}$$

where R_t is the recruitment at year *t* as a result of the spawning stock at the previous time, B_0 is virgin biomass and μ_3 and μ_4 are parameters. Based on Clark (1976) and Bjorndal (1988), the dynamic interactions among recruitment, fish stock, fishing mortality and natural mortality are expressed by a delay-difference equation of the form

$$B_{t+1} = (B_t - h_t)e^{\delta_t} + R_t \tag{17}$$

where h_t is harvest at time *t* and

$$\delta_t = g_t - m \tag{18}$$

for g_t the instantaneous net growth rate and *m* the natural mortality rate. We assume that $t = 0$ corresponds to the time of recruitment of the first cohort. At any time *t* the total biomass of the cohort is

$$B_t = N_t w_t \tag{19}$$

where N_t is the number of fish of the cohort alive at time t and w_t is the average weight of fish at age t. The conversion between fish numbers and fish weight is obtained from the growth in length and length-weight relationship, based on the von Bertalanffy formula (1938) given by

$$l_t = l_\infty \left[1 - e^{-k(t-t_0)}\right] \tag{20}$$

where l_∞ defines an asymptotic or maximum body size, k and is called the Brody growth coefficient and defines growth rate toward the maximum, and t_0 shifts the growth curve along the age axis to allow for apparent nonzero body length at age zero. The length-weight relationship is

$$w_t = w_\infty \left[1 - e^{-k(t-t_0)}\right]^{b_i} \tag{21}$$

where w_∞ is maximum weight.

Model Output

After including random variation in relevant variables and a Brownian motion in terms of the biomass it is not possible to find a solution to equation (15) analytically. Instead, we solve with numerical methods in continuous time using an extended perturbation method (Grafton, et al. 2006). Model output is obtained using *MAPLE 10.0*

Parameters

		Parameter	Value
1a: Western and Central Pacific big eye tuna fishery (Beverton Holt model)			
1	Stock recruitment relationship	μ_3	0.11
2	Stock recruitment relationship	μ_4	$1*10^{-6}$
3	Virgin biomass *(tons)*	S_0	800 000
4	Constant term of harvest function	α_0	0.03
5	Share parameter for effort in the harvest function	α	0.6
6	Share parameter for biomass in the harvest function	β	0.6
7	Parameter for net growth (mortality & weight increase)	ψ	-0.06
8	Parameter of for growth (mortality & weight increase)	χ	0.58
9	Fish price *(AUS$'000/tons)*	p	6.0
10	Costs share parameter per AUS$ 1 revenue	γ^0	0.08
11	Costs share parameter per one unit of effort *($/hook)*	γ^1	3.5
1b: Western and Central Pacific yellow fin tuna fishery (Beverton Holt model)			
1	Stock recruitment relationship	μ_3	0.12
2	Stock recruitment relationship	μ_4	$2*10^{-7}$
3	Virgin biomass *(tons)*	S_0	3 200 000
4	Constant term of harvest function	α_0	2.2
5	Share parameter for effort in the harvest function	α	0.9
6	Share parameter for biomass in the harvest function	β	0.23
7	Parameter for net growth (mortality & weight increase)	ψ	-0.06
8	Parameter for net growth (mortality & weight increase)	χ	0.58
9	Fish price *(AUS$'000/tons)*	p	2.5
10	Costs share parameter per AUS$ 1 revenue	γ^0	0.15
11	Costs share parameter per one unit of effort *(AUS$'000/day)*	γ^1	25

1c: Australian Northern Prawn fishery (Ricker model)			
1	Virgin spawning biomass *(tons)*	$\tilde{S}_0$	1600
2	Spawning stock recruitment parameter	α_1	60
3	Spawning stock- recruitment parameter	β_1	-0.15
4	Recruitment- spawning stock parameter	α_2	0.05
5	Recruitment- spawning stock parameter	β_2	0.4
6	Natural mortality rate	m	0.045
7	Proportion of the total female stock	ϕ	0.5
8	Fishing function parameters	α_3	20
9	Fishing function parameters	β_3	0.4
10	Catchability rate of one unit fishing effort (*CPUE(kg/day))*		$8*10^{-2}$
11	The initial price *(AUS$'000/tons)*	P_0	22
12	Costs share parameter per AUS$ 1 revenue	γ^0	0.1
13	Costs share parameter per one unit of effort *(AUS$'000/day)*	γ^1	5

1d: Australian Orange roughy (Beverton Holt model)			
1	Stock recruitment relationship	μ_3	0.16
2	Stock recruitment relationship	μ_4	$3*10^{-5}$
3	Virgin biomass *(tons)*	S_0	30 000
4	Constant term of harvest function	α_0	0.072
5	Share parameter for effort in the harvest function	α	0.67
6	Share parameter for biomass in the harvest function	β	0.42
7	Parameter of net growth (mortality & weight increase)	ψ	-0.095
8	Parameter of net growth (mortality & weight increase)	χ	0.3
9	Fish price *(AUS$'000/tons)*	p	4.7
10	Costs share parameter per AUS$ 1 revenue	γ^0	0.33
11	Costs share parameter per one unit of effort *(AUS$'000/day)*	γ^1	0.43

References

L. von Bertalanffy, *Human Biology* **10**, 1810-213 (1938).

R. J. H. Beverton, S. J. Holt, *On the Dynamics of Exploited Fish Populations* (Her Majesty’s Stationery Office, 1957).

T. Bjorndal, *J. Environ. Econ. Management* **15**, 9-29 (1988).

C. W. Clark, *Mathematical Bioeconomics* (Wiley, New York, 1976).

R. Q. Grafton, T. Kompas, P. V. Ha, *Econ. Record* **82**, 469-480 (December 2006).

T. J. Quinn II, R. B. Deriso, *Quantitative Fish Dynamics* (Oxford University Press, New York, 1999).