Science Supporting Online Material

To Hedge or Not to Hedge Against an Uncertain Climate Future

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Materials and Methods

We used a modified version of the DICE-99 economic/emissions model constructed by Nordhaus and Boyer (*S1*). The DICE-99 model begins with a time-dependent objective function designed to reflect global welfare, denoted by *W*, as the discounted value of future utility:

$$W = \sum_t U[c(t), L(t)] / R(t) \tag{1a}$$

with utility function *U*[--] depending on per capita consumption $c(t)$ of some aggregate product, population $L(t)$, and a discount factor $R(t)$; *t* indexes time. We chose

$$U[c(t),L(t)] = [L(t)] \ln[c(t)]$$

so that the elasticity of the marginal utility of consumption, a measure of relative aversion to risk, would equal unity and utility in consumption would be convex. In addition, DICE-99 lets

$$R(t) = \prod_{v=0}^{t} [1 + \rho(v)] \tag{1b}$$

so that $\rho(t)$, the social rate of discount (a measure that includes, according the Ramsey rule of discounting, the pure rate of time preference—impatience for future consumption—and a reflection of the marginal utility value of changing per consumption) can change over time. Indeed, many have argued that it should decline over time, and they reflect their argument by letting

$$\rho(t) = \rho(0)\, [\exp(-g^{\rho}\, t)] \tag{1c}$$

with $g^{\rho} > 0$. In this Policy Forum, the pure rate of time preference is set equal to zero. With an elasticity of marginal utility equal to unity, the social discount rate is simply the endogenously determined rate of annual growth of per capita consumption. The equation of motion for population is also specified to accommodate changes in rate of growth over time by letting

$$L(t) = L(0) \cdot \exp \int_0^t g^{\text{pop}}(v) dv$$

with

$$g^{\text{pop}}(t) = g^{\text{pop}}(0) \exp[(-\delta^{\text{pop}})\, t]$$

Setting the parameter $\delta^{\text{pop}} > 0$ captures a systematic decline in the rate of future growth of population.

Economic activity is the source of goods for consumption; but it is also the source of the emissions that drive climate change, as well as investment that drives future growth. The approach adopted by Nordhaus and Boyer represents aggregate global economic activity (gross world product denoted in this Policy Forum as GWP) in a Cobb-Douglas formulation:

$$Q(t) = \Omega(t)[1 - b(t)\mu(t)]\ [A(t)\ K(t)^{\gamma} L(t)^{1-\gamma}] \tag{2}$$

The left-hand side of Eq. 2 denotes net economic output at time t by $Q(t)$. The first term on the right-hand side of Eq. 2 depicts economic damages attributed to climate change by $\Omega(t)$; details underlying its specification will be discussed shortly. The second term on the right-hand side reflects the economic cost of reducing emissions that are derived from the burning of fossil fuel by the industrial sector. Details of this term will also be provided in due course. The third term, finally, represents a production function in capital, denoted $K(t)$, and labor (population). The parameter γ reflects the elasticity of gross aggregate output with respect to the capital stock. The mirror elasticity for labor is given by $(1 - \gamma)$ so that production displays constant returns to scale. Neutral technological change (the ability to squeeze more output from the same amounts of capital and labor) is captured by the $A(t)$ term whose equations of motion are given by $A(t) = A(0) \exp \int_0^t g^A(v) dv$ and

$g^{A}(t) = g^{A}(0) \exp[(-\delta^{A})t]$, respectively.

The pace of technological change can, therefore, decay or accelerate over time, just as the rate of growth of population does.

Net economic output is, in every year, allocated between consumption, $C(t)$, and investment, $I(t)$, according to

$$Q(t) = C(t) + I(t) \tag{3}$$

This simple accounting identity allows per capita consumption to be defined by

$$c(t) = C(t)/L(t)$$

It also allows the equation of motion for the capital stock to be

$$K(t) = (1 - \delta_K)[K(t-1)] + I(t)$$

where $\delta_K > 0$ is the rate of depreciation. Moreover, $K(0)$ fixes a critical initial condition—the stock of capital at time 0.

Emissions of carbon dioxide from industrial sources, denoted $E(t)$, are modeled according to

$$E(t) = [1 - \mu(t)]\,[\sigma(t)]\,[(t)\,K(t)^{\gamma}\,\mathrm{L}(t)^{1-\gamma}] \tag{4}$$

The last term on the right-hand side is a scale factor based on gross economic activity from Eq. 2. The second term relates gross output to gross emissions representing the carbon intensity of production by $\sigma(t)$. This intensity can change with time, so

$$g^{\sigma}(t) = g^{\sigma}(0)\exp[-\delta^{\sigma}(t)]$$

with

$$\sigma(t) = \sigma(t-1)/[1 + g^{\sigma}(t)]$$

and $\sigma(0)$ again fixes an initial condition. The first term, finally, provides a control handle on the level of emissions. The $\mu(t)$ factor can cause net emissions to fall below gross emissions, but emissions reductions are not free. The same factor, modified by $b(t)$, worked in Eq. 2 to reduce net output. Reducing emissions in time t can therefore reduce either consumption or investment in time t, or both.

Equation 2 also allows net economic output to be affected through $\Omega(t)$ by damage that could be attributed to climate change. To relate this damage to changes in global mean temperature, denoted relative to 1990 levels by $T(t)$, DICE-99 defines

$$\Omega(t) = 1/[1 + D(t)]$$

where damages are taken to be quadratic in temperature

$$D(t) = \theta_1\,T(t) + \theta_2\,T(t)^2 \tag{5}$$

The θ_i parameters are both positive, so that damages increase with temperature at an increasing rate.

All that remains at this point is to explain how emissions $E(t)$ are translated into changes in global mean temperature. Atmospheric concentrations of carbon dioxide, denoted $M_A(t)$, provide the critical link. Nordhaus and Boyer adopt a three-equation formulation that reflects the interactions between atmospheric concentrations,

concentrations in the biosphere and upper oceans [$M_{UP}(t)$], and concentrations in the lower oceans [$M_{LO}(t)$]:

$$M_A(t) = E(t-1) + \phi_{11} M_A(t-1) + \phi_{21} M_{UP}(t-1);$$

$$M_{UP}(t) = \phi_{22} M_{UP}(t-1) + \phi_{12} M_A(t-1) + \phi_{32} M_{LO}(t-1); \text{ and}$$

$$M_{LO}(t) = \phi_{33} M_{LO}(t-1) + \phi_{23} M_{UP}(t-1).$$

Atmospheric concentrations are translated into atmospheric forcing according to

$$F(t) = \eta \{\ln[M_A(t)/M_{AT}^{PI}]/\ln(2)\} + O(t) \tag{6}$$

where η is a forcing constant and $O(t)$ reflects forcing from concentrations of other gases that are taken to be independent of economic activity. Finally, increases in global mean temperature above 1990 levels are related to this forcing by another two-equation system that also tracks $T_{LO}(t)$, the temperature of the lower ocean:

$$T(t) = T(t-1) + \alpha_1\{F(t) - \alpha_2[\mathrm{T}(t-1)] - \alpha_3[T(t-1) - T_{LO}(t-1)]\} \tag{7}$$

where

$$T_{LO}(t) = T_{LO}(t-1) + \alpha_4[T(t-1) - T_{LO}(t-1)]$$

Estimates of η, λ, the ϕ_{ij}, the α_i, and initial conditions for T, T_{LO}, M_A, M_{UP}, and M_{LO} are drawn from the scientific literature. A concise description of the calibration process can be found in Nordhaus and Boyer [(*S1*), pp. 57–67].

In DICE-99, $\alpha_3 = 0.44$ and $\alpha_4 = 0.002$. We calibrated the other two parameters to reflect the wide range of climate sensitivities drawn from reference (*S2*) and displayed as a CDF in Fig. 1 of this Policy Forum. Andronova and Schlesinger produced the CDF by using 80,000 realizations of the climate noise, defined as a residual between the simulated (forced) temperature change and the observed temperature departure. A simple atmosphere/ocean model was used to drive temperature response to 16 different radiative forcing models. The radiative forcing models, used to simulate temperature anomalies, represented multiple combinations of individual forcings from greenhouse gases (including CO_2, CFC-11, CFC-12, N_2O, CH_4); tropospheric ozone forcing; anthropogenic and natural sulfate aerosol forcing (direct and indirect); forcing due to variation in the solar constant; and volcanic forcing. Realizations of the climate noise were constructed by using the bootstrap resampling method, which was applied 5000 times to the residual obtained for each radiative forcing model. Each individual value of the climate sensitivity was estimated by minimizing the root-mean-squared error between the simulated and observed temperatures.

Returning now to the DICE-99 model, parameter α_2 is the climate sensitivity and parameter α_1 reflects the inverse of the thermal capacity of the atmospheric layer and the upper oceans. Optimizing for prescribed radiative forcing, including greenhouse gases and anthropogenic sulfate aerosols, produced the appropriate calibrations for a discrete representation of the Andronova and Schlesinger (*S2*) CDF for climate sensitivity; they are reported in Table S1. Calculations for α_1 were made for the prescribed radiative forcing, which included greenhouse gases and anthropogenic sulfate aerosol, but they displayed nonmonotonic behavior between 1°C and 1.2°C, i.e., small values of the thermal capacity were inconsistent with the observations. As a result, 1.5°C was the smallest climate sensitivity included in the discrete version of the CDF.

When these estimates were embedded in DICE-99 for an unregulated future, they produced the temperature trajectories displayed in Fig. S1. The trajectories for 1.5° and 2° climate sensitivities fall in the distant future because emissions of greenhouse gases eventually decline of their own accord (scarcity) and because the shorter lags implied by the associated thermal capacity parameters allow these reductions to appear in the temperature record.

Nordhaus and Boyer (*S1*) report results for stabilizing concentrations at roughly 550 ppm (a doubling target, in their parlance) starting in 1995 and assuming a climate sensitivity of 2.9°C; these results are derived from a deterministic framework where the target is known with certainty in 1995. For purposes of corroboration, the two panels of Fig. S2 compare some of their results with deterministic results derived from the model used here (with the modifications noted in the text) for a 3° climate sensitivity and policy beginning in 2005. The trajectories do not match exactly, but they are not so far apart that the comparison undermines the credibility of using the modified model to explore our hedging experiments.

Supporting Text

We initially explored hedging strategies that serve as the first near-term step toward achieving alternative concentration targets; the actual target was assumed to be determined in 2035. Setting near-term policy as if 550 ppm would turn out to be the final target turned out to be robust over a range of final targets (that include doing nothing past 2035) and over the full range of climate sensitivities.

The most revealing comparison considers 550 ppm against a near-term strategy that does nothing until 2035. Figure S3 shows that the discounted GDP that maximizes near-term strategy for the period between 2005 and 2035 depends on climate sensitivity. Figure S3A (for 1.5°) shows that proceeding through 2035 as if 650 ppm would be the best target even if it were known with certainty that nothing would be done past 2035. This is because doing nothing is not optimal in a cost-benefit framework so that doing a little early improves net-benefits, even if you plan to do nothing later. However, 450 ppm is the best choice for the near term if it were known with certainty that 450 ppm would be the final target. Moreover, the only way to get to 400 ppm is to doing something targeted at 400 ppm now. Similar results are displayed in Fig. S3B for 3°; note that 550 ppm is now the best near-term target if nothing will be done past 2035. Figure S3C shows the results for 9°, and 450 ppm emerges as the best near-term target even if nothing will be done past 2035. Notice that the vertical scale is contracted in Fig. S3C; higher climate sensitivities diminish the difference between GDP levels associated with all of the final targets.

Meeting concentration targets would not eliminate uncertainty over temperature change. Fig. S4A shows a correlation between maximum temperature increase and climate sensitivity for different concentration limits; Fig. S4B displays the same information in terms of cumulative probability distributions.

Fig. S5A shows the smallest maximum temperature that could be achieved with immediate mitigation across the range of climate sensitivities. It shows, for example, that a 2º temperature target could not be achieved even if mitigation were initiated in 2005 if climate sensitivity exceeded slightly more than 5°. Fig. S5B shows the same information in terms of the marginal cost of reducing the likelihood of exceeding three different temperature thresholds in terms of trillions of 1995 dollars in discounted world GDP under the assumption that mitigation began in 2005.

The hedging experiment reported in this Policy Forum asked what tax should be chosen for 2005 (to increase at an endogenous rate of interest through 2035) to maximize the expected discounted value of choosing a temperature target in 2035. Four temperature targets (2º, 2.5º, 3º, and 3.5°) were assumed to be equally likely, and Table S1 provided the likelihoods of various climate sensitivities (the true sensitivity also to be revealed in 2035). As displayed in Fig. S6, some combinations involved doing too little in the near-term, so GWP fell in response to "ramping up" downstream mitigation to achieve the prescribed temperature limit. Other combinations involved doing too much in the near-term, so GWP again fell even though mitigation could be turned down after 2035. As shown in Fig. S7, an initial tax of roughly $10 per ton of carbon (about 5¢ for a gallon of gasoline that would grow at the rate of interest over time) balances these two sources of loss to maximize expected GWP.

Table S2 records some deterministic results for the sake of comparison. The first panel displays the carbon taxes that would be imposed in 2005 to achieve the four temperature targets for each of the climate sensitivities identified in the CDF of Table S1. Notice that low sensitivities are associated with taxes that are smaller than $10 per ton, especially for the higher temperature targets. Put another way, the distribution of deterministic taxes straddles the $10 tax that characterizes an optimal hedging strategy. The second panel shows the discounted net benefit of beginning deterministic policies in 2005. Discounted control costs climb as the climate sensitivity rises, and they fall as the temperature target rises. Discounted gross benefits (avoided economic damages) follow the same pattern, though. As a result, net benefits (negative values indicating costs) become less negative with higher temperature targets and higher climate sensitivities (because higher sensitivities mean that more damages are avoided by achieving any given temperature target). The robustness values recorded in Table 1 of the Policy Forum are additional costs that would exaggerate these negative net benefits. They are small by comparison in most cases, but they can also be as high as 40% of the total for the 2º temperature target.

Supporting figures.

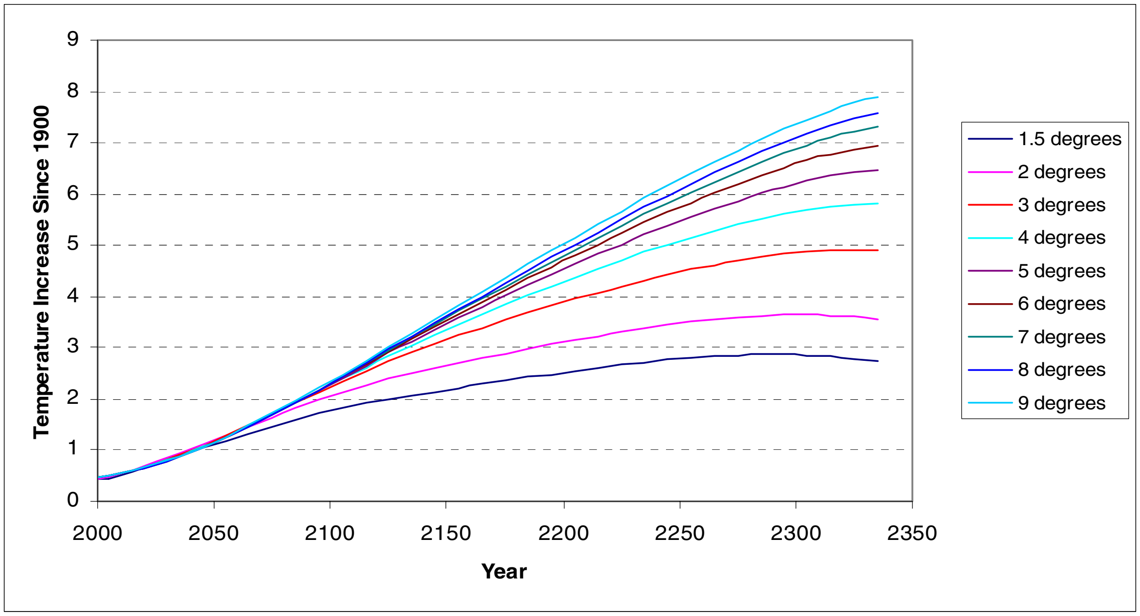


Fig. S1. Temperature trajectories from the DICE-99 baseline runs given alternative climate sensitivities and associated calibrations for the inverse thermal capacity of the atmosphere and the upper oceans. None peak before 2250, even though greenhouse gas emissions fall of their own accord in the distant future.

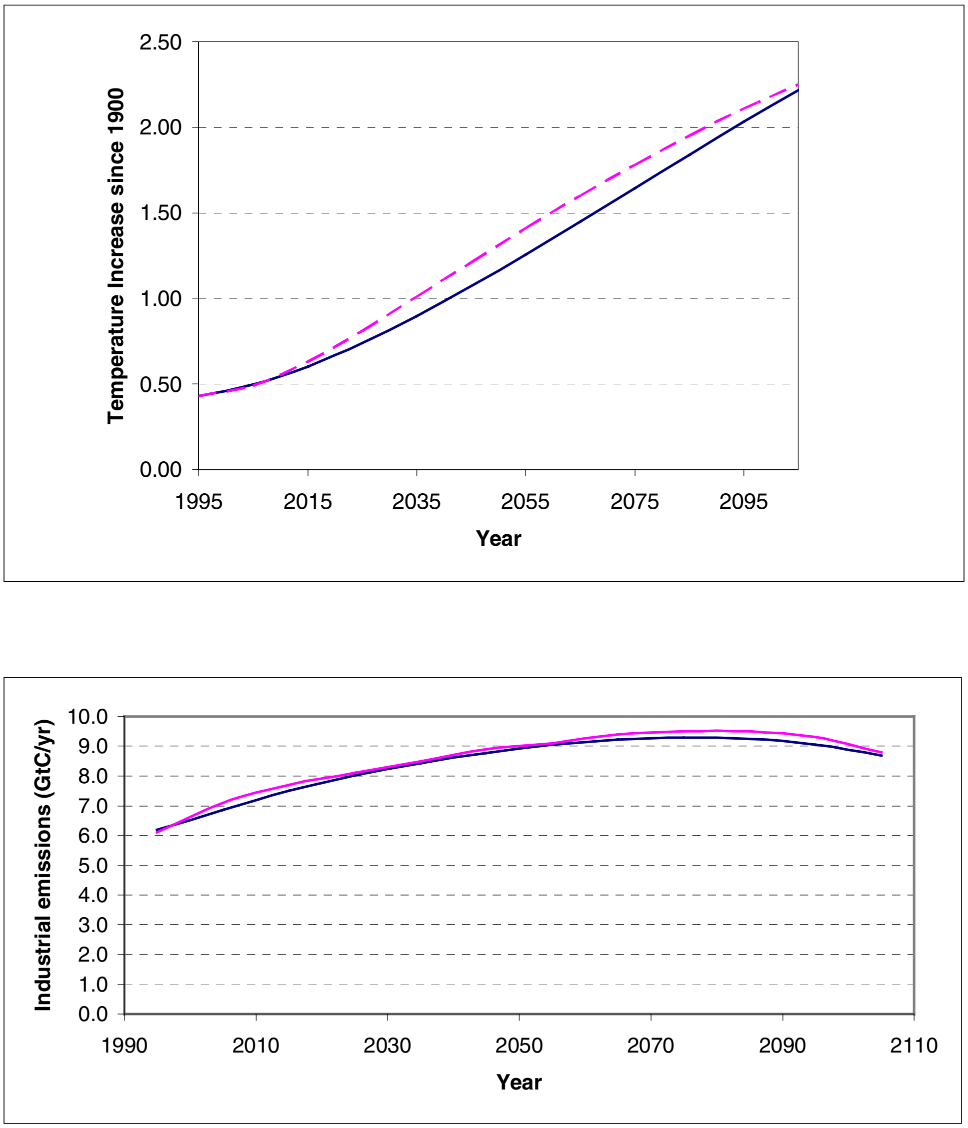
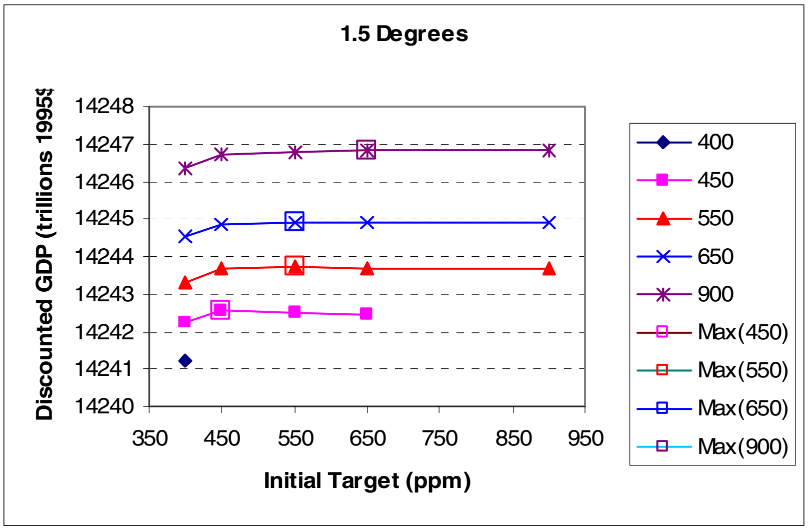


Fig. S2. (**A**) Temperature trajectories along a path that stabilizes concentrations at 550 ppm from Nordhaus and Boyer (*S1*) and this Policy Forum. (**B**) Carbon emissions trajectories along a path that stabilizes concentrations at 550 ppm from Nordhaus and Boyer (*S1*) and this Policy Forum.



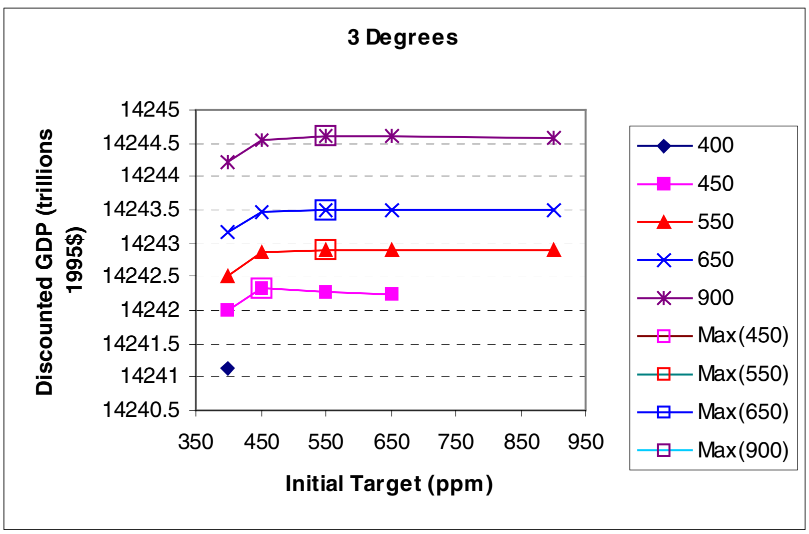


Fig. S3. (**A**) Discounted GWP (in trillions of 1995 dollars) associated with various final mitigation targets for specific initial targets that define near-term policy under the assumption that climate sensitivity is equal to 1.5°; 900 ppm represents an "unregulated" target. The large boxes indicate the initial target that maximizes discounted GDP for the indicated final targets. It is impossible to achieve a 450 ppm target if nothing is done through 2035; and 400 ppm cannot be achieved unless near-term policy is directed toward that target. (**B**) Discounted GWP associated with various final mitigation targets for specific initial targets that define near-term policy under the assumption that climate sensitivity is equal to 3°; 900 ppm represents an "unregulated" target. The large boxes indicate the initial target that maximizes discounted GDP for the indicated final targets. It is impossible to achieve a 450 ppm target if nothing is done through 2035; and 400 ppm cannot be achieved unless near-term policy is directed toward that target.

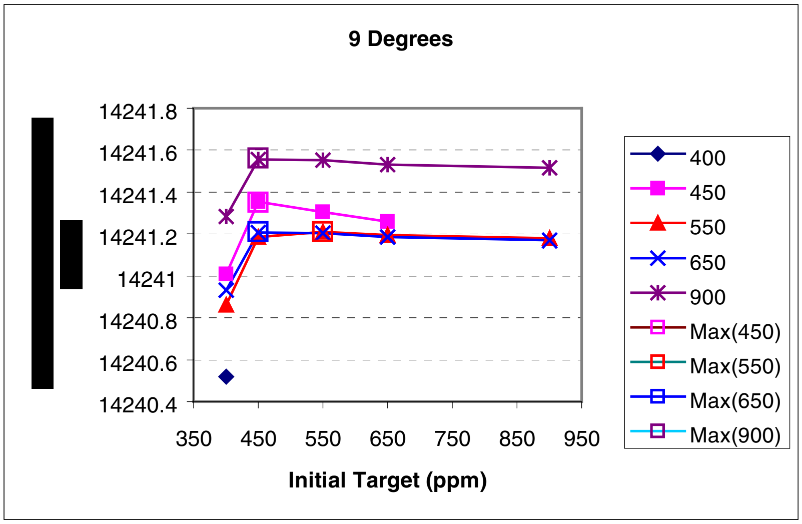
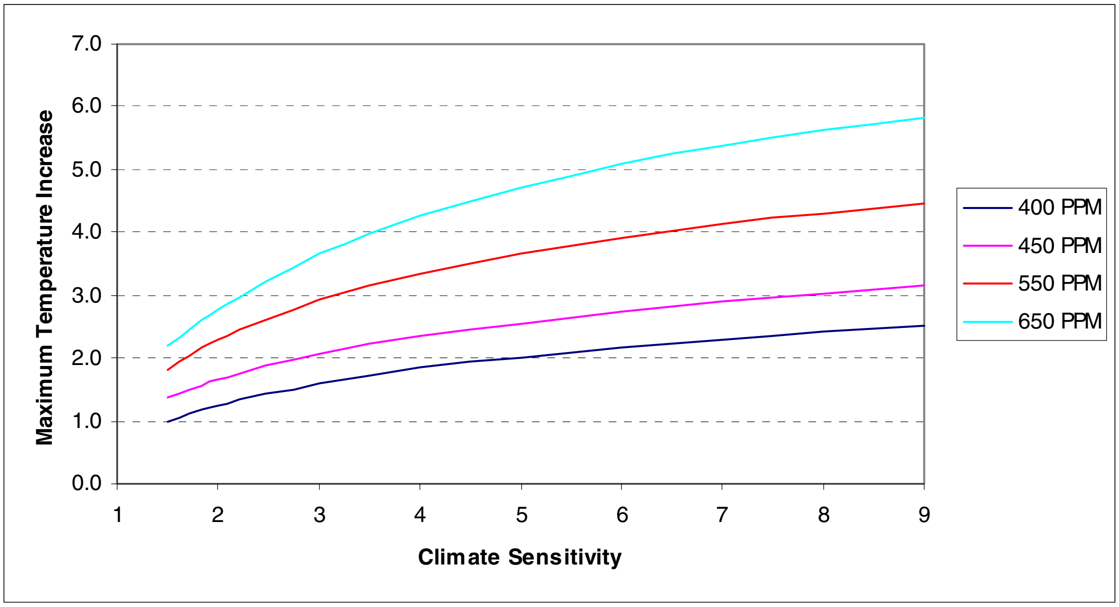


Fig. S3. (**C**)**.** Discounted GWP (in trillions of 1995 dollars) associated with various final mitigation targets for specific initial targets that define near-term policy under the assumption that climate sensitivity is equal to 9°; 900 ppm represents an "unregulated" target. The scale on the vertical axis is different, but the large boxes still indicate the initial target that maximizes discounted GDP for the indicated final targets. It is impossible to achieve a 450 ppm target if nothing is done through 2035; and 400 ppm cannot be achieved unless near-term policy is directed toward that target.



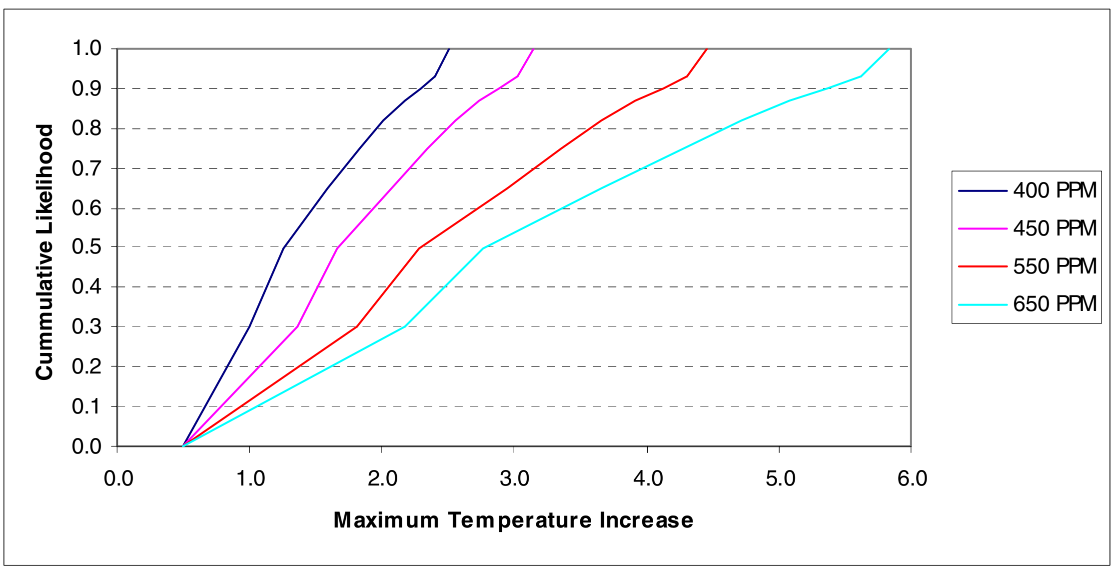
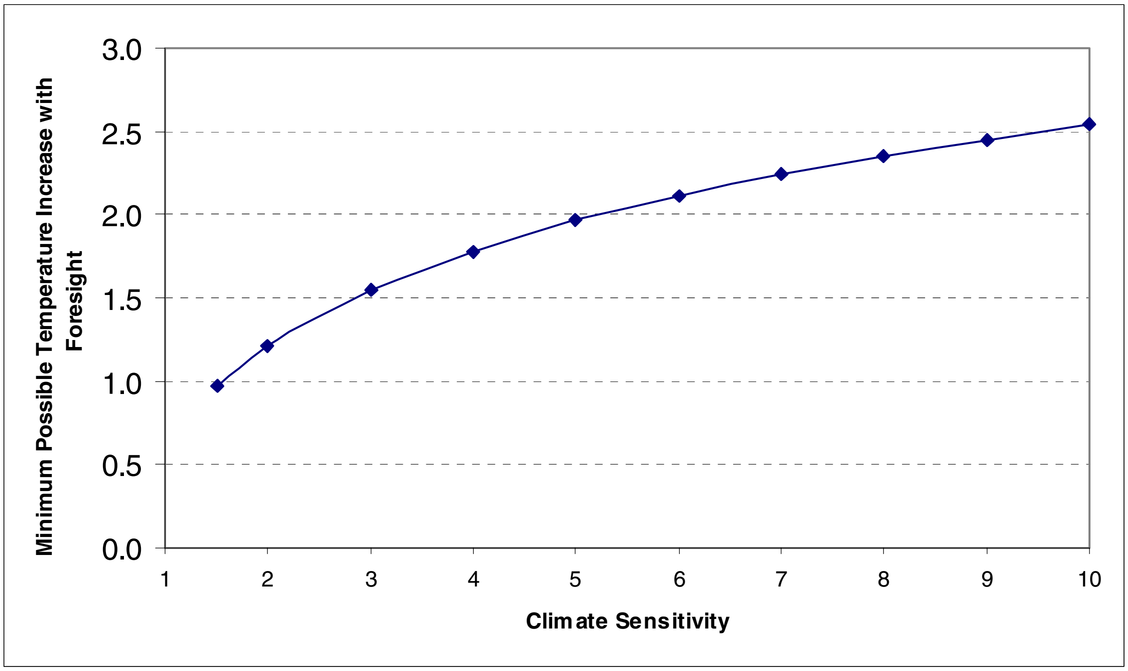


Fig. S4. (**A**) Maximum temperatures associated with alternative concentration targets across the range of climate sensitivities. (**B**) Cumulative probability distributions for the maximum temperatures associated with alternative concentration targets.



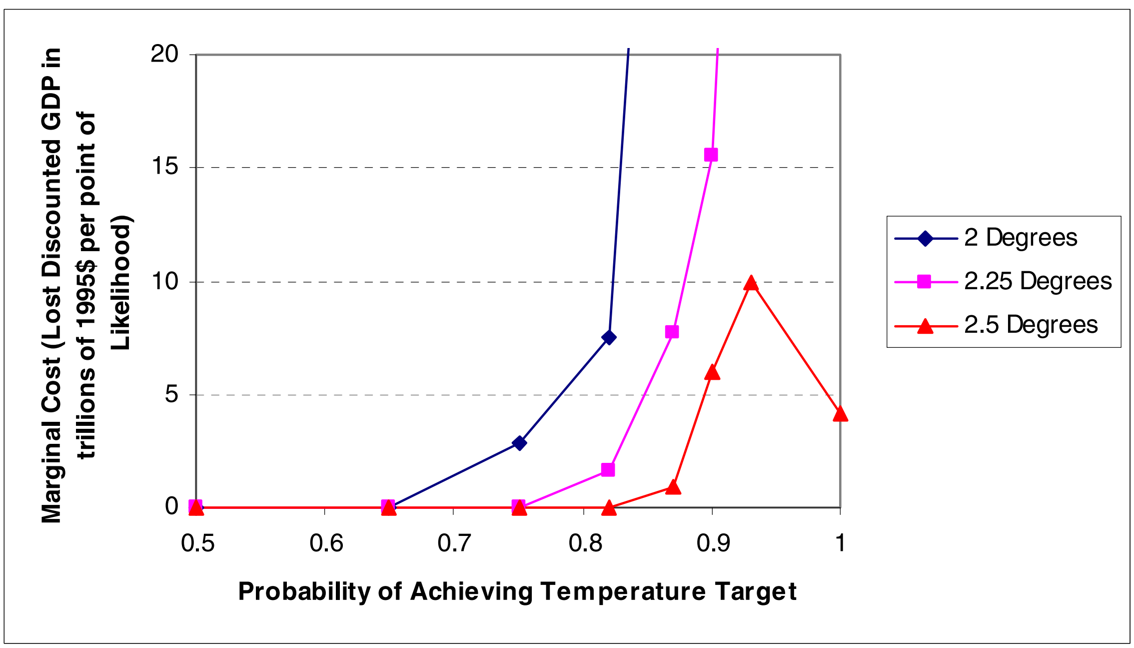


Fig. S5. (**A**) Smallest maximum temperature increases that could be achieved with immediate mitigation beginning in 2005. (**B**) The marginal cost of the probability of achieving alternative temperature targets with immediate mitigation beginning in 2005.

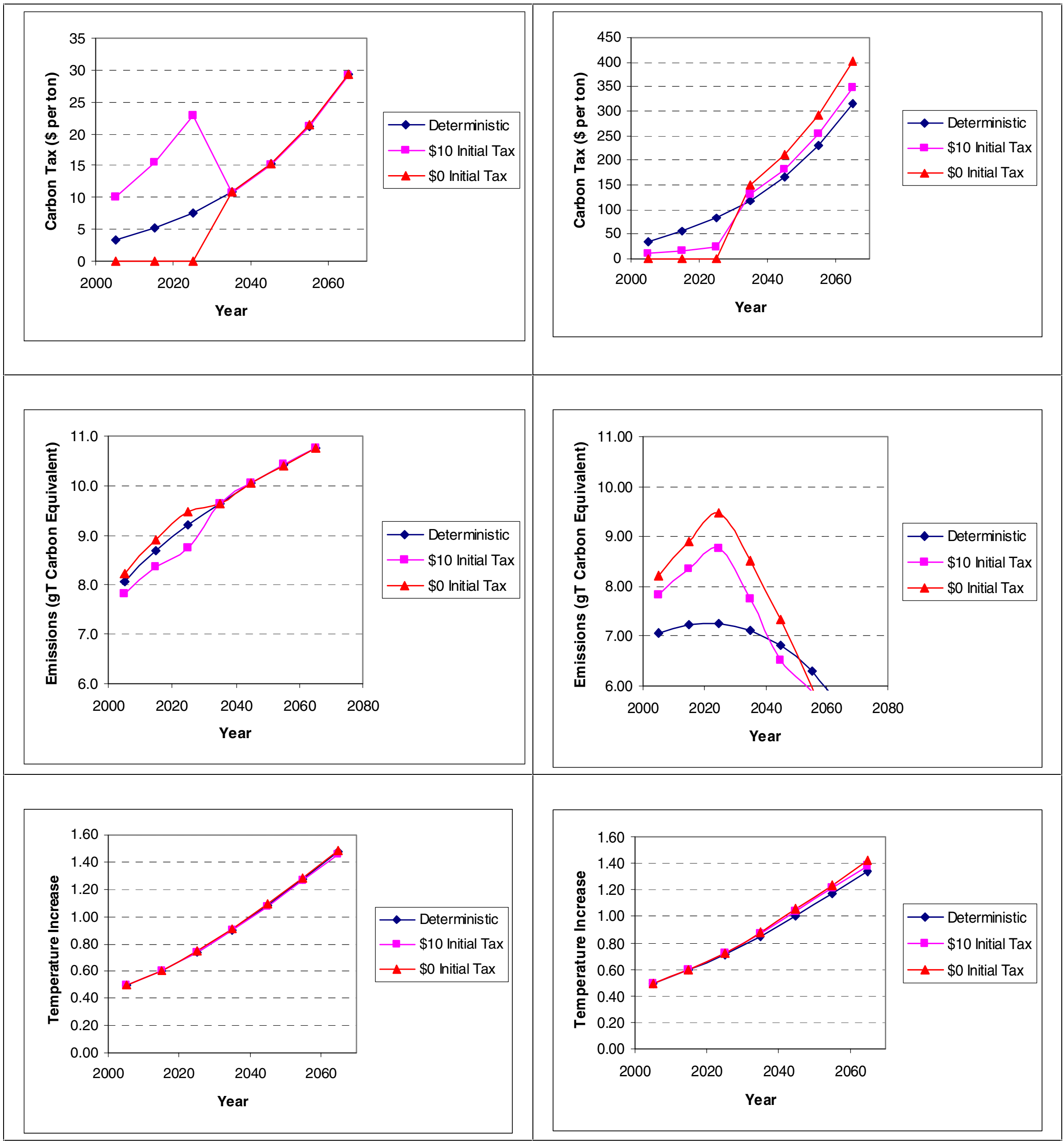


Fig. S6. Results of selected runs are displayed here by comparing the deterministic runs directed at a 3º temperature target with two near-term policies that are "corrected" in 2035. (Left) the case where the target is pursued in a world described by a 3º climate sensitivity. The "do-nothing" policy does too little in the near term (taxes too low and emissions too high through 2035), but the $10 initial tax does too much; nonetheless, the temperature trajectories are difficult to distinguish. (Right) the case where the same 3º temperature target is pursued in a 6º climate sensitivity world. The "do-nothing" policy is now too low (and so too high after 2035), but the $10 tax is also too low in the near term.

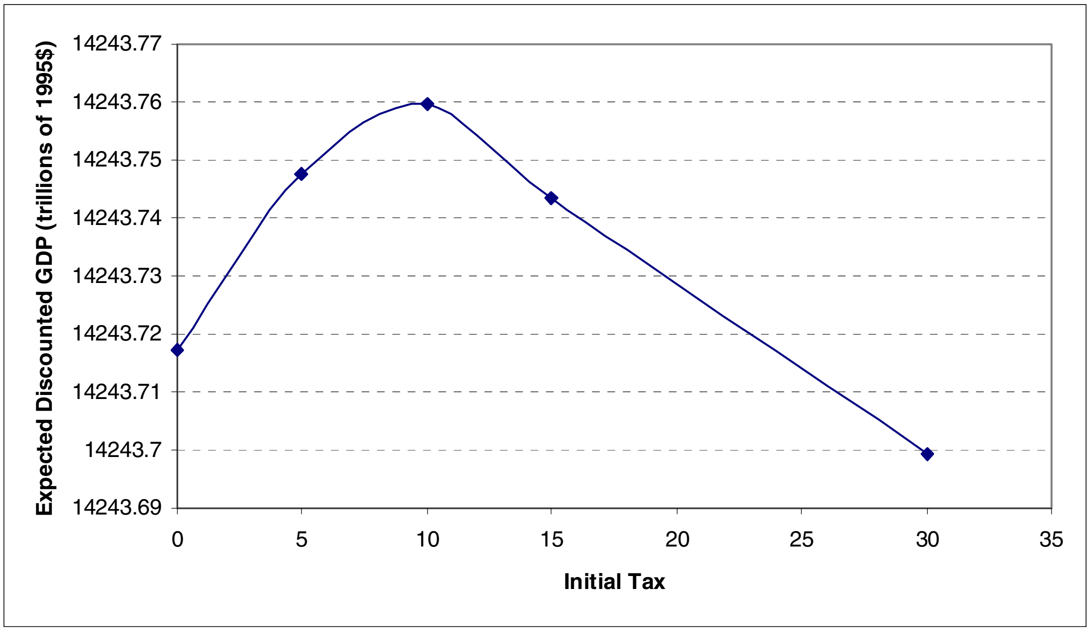


Fig. S7. The difference between the expected discounted GWP of implementing various near-term policy (by prescribing an initial carbon tax and letting it grow at the rate of interest through 2035) is compared with the expected GWP of doing nothing for the first 30 years. In every case, midcourse corrections are made in 2035 so that the expected value calculation, conducted in 2005, reflects equally likely temperature targets and the CDF of climate sensitivity displayed in Fig. 1 of this Policy Forum and Table S1.

Table S1. The discrete representation of the cumulative distribution function for climate sensitivity from Andronova and Schlesinger (*S2*) expressed in terms of climate sensitivity (the α_2 parameter in Eq. 7), the likelihood, and the associated inverse thermal capacity (the α_1 parameter in Eq. 7) that optimized for the prescribed radiative forcing.

Climate sensitivity (α_2)	**Likelihood**	**Corresponding** α_1 **calibration**
1.5°	0.30	0.065742
2°	0.20	0.027132
3°	0.15	0.014614
4°	0.10	0.011550
5°	0.07	0.010278
6°	0.05	0.009589
7°	0.03	0.009157
8°	0.03	0.008863
9°	0.07	0.008651

Table S2. Deterministic results for policies designed and implemented in 2005 to achieve specific maximum temperature targets with perfect knowledge about the climate sensitivity. Black cells indicate temperature targets that cannot be achieved for the specified climate sensitivities. Purple cell indicates a case where the decarbonization constraint binds even in the deterministic case.

Climate sensitivity (α_2)	*Taxes in 2005 (1995$ per ton of carbon) for a temperature target of*			
	2°	**2.5°**	**3°**	**3.5°**
1.5	4.88	1	0	0
2	16.3	5.22	1.6	0.27
3	35.65	16.55	7.81	3.63
4	54.29	25.6	13.33	7.12
5	77.55	34.78	18.49	10.3
6		44.07	23.43	13.31
7		53.79	28.2	16.19
8		64.35	32.79	18.95
9		76.97	37.48	21.56

Climate sensitivity (α_2)	*Discounted net benefits (trillions of 1995$) for a temperature target of*			
	2°	**2.5°**	**3°**	**3.5°**
1.5	–2.56	–0.38	0	0
2	–2.89	–2.21	–0.82	0.06
3	–2.39	–1.88	–1.68	–1.27
4	–2.09	–1.39	–1.21	–1.17
5	–2.07	–1.13	–0.89	–0.83
6		–0.98	–0.64	–0.58
7		–0.91	–0.47	–0.37
8		–0.9	–0.32	–0.25
9		–0.98	–0.25	–0.13

References and Notes

S1. W.D. Nordhaus, J. Boyer, *Warming the World—Economic Models of Global Warming* (MIT Press, Cambridge, MA, 2001).
S2. N.G. Andronova, M. E. Schlesinger, *J. Geophys. Res.* **106** (D190), 22605 (2001).